

Contents

<i>Preface</i>	vii
1. Calculus on smooth manifolds	1
1.1 The Euclidean structure on the exterior algebra	1
1.2 The star isomorphism on $\Lambda(V)$	2
1.3 The tangent and cotangent bundles of a smooth manifold	4
1.4 The de Rham cohomology groups	6
1.5 Riemannian metrics	11
1.6 Partitions of unity	12
1.7 Orientation and integration	12
2. The Hodge theory of a smooth, oriented, compact Riemannian manifold	19
2.1 The adjoint of d : d^*	19
2.2 The Laplace-Beltrami operator of an oriented Riemannian manifold	21
2.3 Harmonic forms and the Hodge Isomorphism Theorem . .	22
3. Complex manifolds	27
3.1 Conjugations	27
3.2 Tangent bundles on a complex manifold	28
3.3 Cotangent bundles on complex manifolds	31
3.4 The standard orientation of a complex manifold	33
3.5 The quasi complex structure	34
3.6 Complex-valued forms	37
3.7 Dolbeault and Bott-Chern cohomology	40

4.	Hermitean linear algebra	43
4.1	The exterior algebra on $V_{\mathbb{C}}^*$	43
4.2	Bases	44
4.3	Hermitean metrics	45
4.4	The inner product and the \star operator on the complexified exterior algebra $\Lambda_{\mathbb{C}}(V_{\mathbb{C}}^*)$	48
4.5	The Weil operator	50
5.	The Hodge theory of Hermitean manifolds	51
5.1	Hermitean metrics on complex manifolds	51
5.2	The Hodge theory of a compact Hermitean manifold . . .	53
6.	Kähler manifolds	57
6.1	The Kähler condition	57
6.2	The fundamental identities of Kähler geometry	61
6.3	The Hodge Decomposition for compact Kähler manifolds	66
6.4	Some consequences	69
7.	The Hard Lefschetz Theorem and the Hodge-Riemann Bilinear Relations	71
7.1	Hodge structures	72
7.2	The cup product with the Chern class of a hyperplane bundle	74
7.3	The Hard Lefschetz Theorem and the Hodge-Riemann Bilinear Relations	76
7.4	The Weak Lefschetz Theorem	80
8.	Mixed Hodge structures, semi-simplicity and approximability	83
8.1	The mixed Hodge structure on the cohomology of complex algebraic varieties	83
8.2	The Semi-simplicity Theorem	85
8.3	The Leray spectral sequence	87
8.4	The Global Invariant Cycle Theorem	88
8.5	The Lefschetz Theorems and semi-simplicity	89
8.6	Approximability for the space of primitive vectors	93

<i>Bibliography</i>	99
---------------------	----

<i>Index</i>	101
--------------	-----