Contents

Preface ix

Chapter 1. Pseudodifferential Operators with Mildly Regular Symbols 1
§1. Spaces of continuous functions 3
§2. Operator estimates on L^p, $\mathfrak{h}^1$, and bmo 17
§3. Symbol classes and symbol smoothing 31
§4. Operator estimates on Sobolev-like spaces 37
§5. Operator estimates on spaces $C^{(\lambda)}$ 44
§6. Products 54
§7. Commutator estimates 58
§8. Operators with Sobolev coefficients 61
§9. Operators with double symbols 63
§10. The CRW commutator estimate 75
§11. Operators with vmo coefficients 78
§12. Estimates on a class of Besov spaces 82
§13. Operators with coefficients in a function algebra 86
§14. Some BKM-type estimates 88
§15. Variations on an estimate of Tumanov 92
§16. Estimates on Morrey-type spaces 94

Chapter 2. Paradifferential Operators and Nonlinear Estimates 101
§1. A product estimate 105
§2. A commutator estimate 106
§3. Some handy estimates involving maximal functions 108
§4. A composition estimate 110
§5. More general composition estimate 112
§6. Continuity of $u \mapsto f(u)$ on $H^{1,p}$ 113
§7. Estimates on $F(u) - F(v)$ 116
§8. A pseudodifferential operator estimate 118
§9. Paradifferential operators on the spaces $C^{(\lambda)}$ 120
§A. Paracomposition 125
§B. Alinhac's lemma 132

Chapter 3. Applications to PDE 135
§1. Interior elliptic regularity 137
§2. Some natural first-order operators 148
§3. Estimates for the Dirichlet problem 155
§4. Layer potentials on $C^{1,\omega}$ surfaces 159

§5. Parametrix estimates and trace asymptotics 173
§6. Euler flows on rough planar domains 178
§7. Persistence of solutions to semilinear wave equations 183
§8. Div-curl estimates 186
§9. Harmonic coordinates 194
§10. Riemannian manifolds with bounded Ricci tensor 202
§11. Propagation of singularities 205

Chapter 4. Layer Potentials on Lipschitz Surfaces 217
§1. Cauchy kernels on Lipschitz curves 218
§2. The method of rotations and extensions to higher dimensions 228
§3. The variable-coefficient case 230
§4. Boundary integral operators 235
§5. The Dirichlet problem on Lipschitz domains 241
§A. The Koebe-Bieberbach distortion theorem 246

Bibliography 249

List of Symbols 255

Index 257