CONTENTS

Introduction ..................................................... 1

I. Gauss’ circle problem and Pólya’s random walks on lattices 5
   I.A. The circle problem ........................................... 5
   I.B. Pólya’s recurrence theorem ................................. 7

II. Free products and free groups ................................. 17
   II.A. Free products of groups ................................. 17
   II.B. The Table-Tennis Lemma (Klein's criterion) and examples
         of free products ........................................... 25

III. Finitely-generated groups .................................. 43
    III.A. Finitely-generated and infinitely-generated groups 43
    III.B. Uncountably many groups with two generators (B.H.
           Neumann's method) ....................................... 60
    III.C. On groups with two generators ........................ 68
    III.D. On finite quotients of the modular group ............ 71

IV. Finitely-generated groups viewed as metric spaces ......... 75
    IV.A. Word lengths and Cayley graphs ....................... 75
    IV.B. Quasi-isometries ........................................ 84

V. Finitely-presented groups .................................. 117
    V.A. Finitely-presented groups ............................... 117
    V.B. The Poincaré theorem on fundamental polygons ........ 135
    V.C. On fundamental groups and curvature in Riemannian
         geometry .................................................... 145
    V.D. Complement on Gromov’s hyperbolic groups .......... 148

VI. Growth of finitely-generated groups ....................... 151
    VI.A. Growth functions and growth series of groups ....... 151
    VI.B. Generalities on growth types .......................... 167
    VI.C. Exponential growth rate and entropy .................. 180
VII. Groups of exponential or polynomial growth  187
   VII.A. On groups of exponential growth  187
   VII.B. On uniformly exponential growth  194
   VII.C. On groups of polynomial growth  197
   VII.D. Complement on other kinds of growth  206

VIII. The first Grigorchuk group  211
   VIII.A. Rooted d-ary trees and their automorphisms  211
   VIII.B. The group $\Gamma$ as an answer to one of Burnside's problems  217
   VIII.C. On some subgroups of $\Gamma$  225
   VIII.D. Congruence subgroups  236
   VIII.E. Word problem and non-existence of finite presentations  240
   VIII.F. Growth  248
   VIII.G. Exercises and complements  259

References  265
Index of research problems  295
Appendix: Corrections and Updates  299
Subject index  323