

Contents

PREFACE	ix
AN INVITATION TO ANALYTIC COMBINATORICS	1
Part A. SYMBOLIC METHODS	13
I. COMBINATORIAL STRUCTURES AND ORDINARY GENERATING FUNCTIONS	15
I. 1. Symbolic enumeration methods	16
I. 2. Admissible constructions and specifications	24
I. 3. Integer compositions and partitions	39
I. 4. Words and regular languages	49
I. 5. Tree structures	64
I. 6. Additional constructions	83
I. 7. Perspective	92
II. LABELLED STRUCTURES AND EXPONENTIAL GENERATING FUNCTIONS	95
II. 1. Labelled classes	96
II. 2. Admissible labelled constructions	100
II. 3. Surjections, set partitions, and words	106
II. 4. Alignments, permutations, and related structures	119
II. 5. Labelled trees, mappings, and graphs	125
II. 6. Additional constructions	136
II. 7. Perspective	147
III. COMBINATORIAL PARAMETERS AND MULTIVARIATE GENERATING FUNCTIONS	151
III. 1. An introduction to bivariate generating functions (BGFs)	152
III. 2. Bivariate generating functions and probability distributions	156
III. 3. Inherited parameters and ordinary MGFs	163
III. 4. Inherited parameters and exponential MGFs	174
III. 5. Recursive parameters	181
III. 6. Complete generating functions and discrete models	186
III. 7. Additional constructions	198
III. 8. Extremal parameters	214
III. 9. Perspective	218
Part B. COMPLEX ASYMPTOTICS	221
IV. COMPLEX ANALYSIS, RATIONAL AND MEROMORPHIC ASYMPTOTICS	223
IV. 1. Generating functions as analytic objects	225
IV. 2. Analytic functions and meromorphic functions	229

IV. 3.	Singularities and exponential growth of coefficients	238
IV. 4.	Closure properties and computable bounds	249
IV. 5.	Rational and meromorphic functions	255
IV. 6.	Localization of singularities	263
IV. 7.	Singularities and functional equations	275
IV. 8.	Perspective	286
V.	APPLICATIONS OF RATIONAL AND MEROMORPHIC ASYMPTOTICS	289
V. 1.	A roadmap to rational and meromorphic asymptotics	290
V. 2.	The supercritical sequence schema	293
V. 3.	Regular specifications and languages	300
V. 4.	Nested sequences, lattice paths, and continued fractions	318
V. 5.	Paths in graphs and automata	336
V. 6.	Transfer matrix models	356
V. 7.	Perspective	373
VI.	SINGULARITY ANALYSIS OF GENERATING FUNCTIONS	375
VI. 1.	A glimpse of basic singularity analysis theory	376
VI. 2.	Coefficient asymptotics for the standard scale	380
VI. 3.	Transfers	389
VI. 4.	The process of singularity analysis	392
VI. 5.	Multiple singularities	398
VI. 6.	Intermezzo: functions amenable to singularity analysis	401
VI. 7.	Inverse functions	402
VI. 8.	Polylogarithms	408
VI. 9.	Functional composition	411
VI. 10.	Closure properties	418
VI. 11.	Tauberian theory and Darboux's method	433
VI. 12.	Perspective	437
VII.	APPLICATIONS OF SINGULARITY ANALYSIS	439
VII. 1.	A roadmap to singularity analysis asymptotics	441
VII. 2.	Sets and the exp–log schema	445
VII. 3.	Simple varieties of trees and inverse functions	452
VII. 4.	Tree-like structures and implicit functions	467
VII. 5.	Unlabelled non-plane trees and Pólya operators	475
VII. 6.	Irreducible context-free structures	482
VII. 7.	The general analysis of algebraic functions	493
VII. 8.	Combinatorial applications of algebraic functions	506
VII. 9.	Ordinary differential equations and systems	518
VII. 10.	Singularity analysis and probability distributions	532
VII. 11.	Perspective	538
VIII.	SADDLE-POINT ASYMPTOTICS	541
VIII. 1.	Landscapes of analytic functions and saddle-points	543
VIII. 2.	Saddle-point bounds	546
VIII. 3.	Overview of the saddle-point method	551
VIII. 4.	Three combinatorial examples	558
VIII. 5.	Admissibility	564
VIII. 6.	Integer partitions	574

VIII. 7. Saddle-points and linear differential equations.	581
VIII. 8. Large powers	585
VIII. 9. Saddle-points and probability distributions	594
VIII. 10. Multiple saddle-points	600
VIII. 11. Perspective	606
Part C. RANDOM STRUCTURES	609
IX. MULTIVARIATE ASYMPTOTICS AND LIMIT LAWS	611
IX. 1. Limit laws and combinatorial structures	613
IX. 2. Discrete limit laws	620
IX. 3. Combinatorial instances of discrete laws	628
IX. 4. Continuous limit laws	638
IX. 5. Quasi-powers and Gaussian limit laws	644
IX. 6. Perturbation of meromorphic asymptotics	650
IX. 7. Perturbation of singularity analysis asymptotics	666
IX. 8. Perturbation of saddle-point asymptotics	690
IX. 9. Local limit laws	694
IX. 10. Large deviations	699
IX. 11. Non-Gaussian continuous limits	703
IX. 12. Multivariate limit laws	715
IX. 13. Perspective	716
Part D. APPENDICES	719
Appendix A. AUXILIARY ELEMENTARY NOTIONS	721
A.1. Arithmetical functions	721
A.2. Asymptotic notations	722
A.3. Combinatorial probability	727
A.4. Cycle construction	729
A.5. Formal power series	730
A.6. Lagrange inversion	732
A.7. Regular languages	733
A.8. Stirling numbers.	735
A.9. Tree concepts	737
Appendix B. BASIC COMPLEX ANALYSIS	739
B.1. Algebraic elimination	739
B.2. Equivalent definitions of analyticity	741
B.3. Gamma function	743
B.4. Holonomic functions	748
B.5. Implicit Function Theorem	753
B.6. Laplace's method	755
B.7. Mellin transforms	762
B.8. Several complex variables	767
Appendix C. CONCEPTS OF PROBABILITY THEORY	769
C.1. Probability spaces and measure	769
C.2. Random variables	771
C.3. Transforms of distributions	772

C.4.	Special distributions	774
C.5.	Convergence in law	776
BIBLIOGRAPHY		779
INDEX		801