

Contents

| | |
|---|-----------|
| Preface | v |
| Introduction | 1 |
| 1 Boundary value problems with mixed and interface data | 13 |
| 1.1 Elliptic boundary value problems | 13 |
| 1.1.1 Differential operators with classical boundary conditions | 13 |
| 1.1.2 The Poisson kernels in the half-space | 18 |
| 1.1.3 Reduction to the boundary | 21 |
| 1.1.4 Examples | 26 |
| 1.2 Mixed and transmission problems | 30 |
| 1.2.1 Mixed problems in weighted edge spaces | 30 |
| 1.2.2 Additional conditions at the interface | 34 |
| 1.2.3 Examples | 42 |
| 1.2.4 Reduction of orders and reduction to the boundary | 45 |
| 1.2.5 Mixed problems in standard Sobolev spaces | 49 |
| 1.3 Problems with several types of interfaces | 52 |
| 1.3.1 Transmission problems with smooth interfaces | 52 |
| 1.3.2 Transmission problems with singular interfaces | 54 |
| 2 Symbolic structures and associated operators | 57 |
| 2.1 Scalar pseudo-differential calculus | 57 |
| 2.1.1 Spaces of symbols and basic operations | 57 |
| 2.1.2 Pseudo-differential operators and Sobolev spaces | 61 |
| 2.1.3 Operators on manifolds | 65 |
| 2.2 Calculus with operator-valued symbols | 72 |
| 2.2.1 Symbols and operators with twisted homogeneity | 73 |
| 2.2.2 Abstract edge spaces | 79 |
| 2.2.3 Elements of the calculus | 87 |
| 2.3 Operators on manifolds with conical exit to infinity | 102 |
| 2.3.1 Symbols with exit behaviour | 102 |
| 2.3.2 Operators globally in the Euclidean space | 106 |
| 2.3.3 Operators on manifolds | 107 |
| 2.3.4 Ellipticity in the scalar case | 111 |
| 2.3.5 Calculus with operator-valued symbols | 112 |
| 2.4 Mellin operators | 120 |
| 2.4.1 The Mellin transform | 120 |
| 2.4.2 Weighted Sobolev spaces | 122 |
| 2.4.3 Degenerate differential operators | 132 |
| 2.4.4 Mellin operators and quantisation | 137 |

| | | |
|----------|--|------------|
| 2.4.5 | A connection between edge-degenerate operators and exit calculus | 146 |
| 2.4.6 | Green operators for conical singularities | 155 |
| 3 | Boundary value problems with the transmission property | 167 |
| 3.1 | Interior and boundary symbols | 167 |
| 3.1.1 | Symbols with the transmission property | 167 |
| 3.1.2 | Operators with the transmission property | 173 |
| 3.1.3 | Green operators on the half-axis | 176 |
| 3.1.4 | Boundary value problems on the half-axis | 180 |
| 3.1.5 | Operators on an interval | 185 |
| 3.2 | The algebra of boundary value problems | 187 |
| 3.2.1 | Global smoothing operators | 188 |
| 3.2.2 | Green operators | 189 |
| 3.2.3 | Boundary value problems | 192 |
| 3.3 | Ellipticity and parametrices | 200 |
| 3.3.1 | Elliptic boundary value problems | 200 |
| 3.3.2 | Parametrices and inverses | 202 |
| 3.3.3 | Parameter-dependent ellipticity | 205 |
| 3.3.4 | Fredholm families and block matrix isomorphisms | 207 |
| 3.4 | The calculus on manifolds with conical exit to infinity | 214 |
| 3.4.1 | Motivation in terms of principal edge symbols | 214 |
| 3.4.2 | Global operators in the half-space | 217 |
| 3.4.3 | Operators on a manifold with conical exit | 221 |
| 3.4.4 | A relation between edge-degenerate families and exit calculus | 224 |
| 4 | Mixed problems in standard Sobolev spaces | 225 |
| 4.1 | Reductions of orders on a manifold with boundary | 225 |
| 4.1.1 | Order reducing symbols in the half-space | 225 |
| 4.1.2 | Actions in Sobolev spaces | 228 |
| 4.1.3 | A relationship with classical Volterra symbols | 230 |
| 4.1.4 | Global reduction of orders | 233 |
| 4.1.5 | General operators with plus/minus-symbols | 235 |
| 4.2 | Mixed elliptic problems | 240 |
| 4.2.1 | Mixed problems for differential operators | 241 |
| 4.2.2 | Ellipticity with additional conditions at the interface | 257 |
| 4.2.3 | The Zarembo problem | 263 |
| 4.2.4 | Jumping oblique derivatives and other examples | 266 |
| 5 | Mixed problems in weighted edge spaces | 271 |
| 5.1 | Mixed problems in edge spaces | 271 |
| 5.1.1 | Basic observations | 271 |
| 5.1.2 | Green symbols | 274 |
| 5.1.3 | The Zarembo problem as an edge problem | 275 |

| | | |
|----------|---|------------|
| 5.2 | Relations between edge and standard Sobolev spaces | 279 |
| 5.2.1 | Spaces on the boundary | 279 |
| 5.2.2 | Edge spaces in the stretched domain | 281 |
| 5.2.3 | A reformulation of mixed problems from standard Sobolev spaces | 283 |
| 5.3 | Elliptic interface conditions | 287 |
| 5.3.1 | Mixed problems in spaces of arbitrary weights | 287 |
| 5.3.2 | Construction of elliptic interface conditions | 289 |
| 5.3.3 | Parametrices and regularity of solutions for the Zaremba problem | 291 |
| 5.3.4 | Jumping oblique derivatives and other examples | 292 |
| 5.4 | Edge calculus, specified to mixed problems | 298 |
| 5.4.1 | Edge amplitude functions | 298 |
| 5.4.2 | Edge-boundary value problems | 301 |
| 5.4.3 | Ellipticity and parametrices | 305 |
| 5.4.4 | Asymptotics of solutions | 306 |
| 6 | Operators on manifolds with conical singularities and boundary | 309 |
| 6.1 | Fuchs type operators and Mellin quantisation | 309 |
| 6.1.1 | Mellin quantisation | 309 |
| 6.1.2 | Kernel cut-off | 323 |
| 6.1.3 | Meromorphic Fredholm families and ellipticity of conormal symbols | 332 |
| 6.1.4 | Green operators | 338 |
| 6.1.5 | Mellin operators with smoothing symbols | 344 |
| 6.1.6 | Operators with holomorphic Mellin symbols | 346 |
| 6.2 | The cone algebra | 355 |
| 6.2.1 | Operators on a compact manifold with conical singularities and boundary | 356 |
| 6.2.2 | Operators on an infinite cone with boundary | 363 |
| 6.3 | Boundary value problems in plane domains | 369 |
| 6.3.1 | The Dirichlet problem in a strip | 369 |
| 6.3.2 | The Neumann and the Zaremba problem in a strip | 373 |
| 6.3.3 | The Dirichlet problem in an angle | 376 |
| 6.3.4 | The Neumann and the Zaremba problem in an angle | 378 |
| 6.4 | Special operators of the cone calculus | 380 |
| 6.4.1 | Reduction of orders | 380 |
| 6.4.2 | Operators on a cone with arbitrary weights at infinity | 385 |
| 6.4.3 | Cone operators with parameters | 389 |
| 6.4.4 | Elliptic regularity for some Schrödinger equation | 391 |

| | | |
|----------|--|-----|
| 7 | Operators on manifolds with edges and boundary | 394 |
| 7.1 | Differential operators on manifolds with edges | 394 |
| 7.1.1 | Edge-degenerate differential operators | 394 |
| 7.1.2 | Weighted edge spaces | 397 |
| 7.1.3 | Edge-boundary value problems as operators in weighted spaces | 404 |
| 7.1.4 | Operators in alternative weighted edge spaces | 406 |
| 7.2 | The edge algebra | 408 |
| 7.2.1 | Edge-degenerate symbols and operator conventions | 409 |
| 7.2.2 | Global smoothing operators | 413 |
| 7.2.3 | Green and smoothing Mellin symbols | 416 |
| 7.2.4 | Edge amplitude functions | 420 |
| 7.2.5 | The edge algebra | 428 |
| 7.2.6 | Ellipticity and reductions of orders | 434 |
| 7.3 | Mellin-edge representations of elliptic operators | 442 |
| 7.3.1 | Decomposition of classical Sobolev spaces | 442 |
| 7.3.2 | Edge decompositions of differential operators | 447 |
| 7.3.3 | Global constructions | 451 |
| 7.3.4 | Edge representation of boundary value problems | 457 |
| 7.3.5 | Relative index results | 460 |
| 7.3.6 | Interface conditions for small weights | 463 |
| 7.4 | The Laplacian in a wedge, and other elliptic operators of the edge calculus | 464 |
| 7.4.1 | The Dirichlet problem in a wedge | 464 |
| 7.4.2 | The Neumann and the Zaremba problem in a wedge | 467 |
| 7.4.3 | Other examples of elliptic edge operators | 468 |
| 8 | Corner operators and problems with singular interfaces | 475 |
| 8.1 | Singular mixed problems and corner manifolds | 475 |
| 8.1.1 | The singular Zaremba problem | 475 |
| 8.1.2 | Operators in edge representation | 477 |
| 8.1.3 | Principal symbols and edge conditions | 479 |
| 8.1.4 | Corner manifolds | 481 |
| 8.2 | Corner operators in spaces with double weights | 484 |
| 8.2.1 | Transformation to a corner boundary value problem | 484 |
| 8.2.2 | Corner spaces with double weights | 486 |
| 8.2.3 | Continuity in corner spaces | 491 |
| 8.2.4 | Holomorphic corner symbols | 495 |
| 8.2.5 | Corner boundary value problems | 497 |
| 8.2.6 | Ellipticity near the corner | 501 |
| 8.3 | Corner edge operators | 503 |
| 8.3.1 | Global corner boundary value problems | 503 |
| 8.3.2 | Ellipticity and parametrices | 508 |
| 8.3.3 | The singular Zaremba problem | 509 |
| 8.3.4 | Remarks | 512 |

| | | |
|-----------|---|------------|
| 8.4 | Cracks with singularities at the boundary | 514 |
| 8.4.1 | Crack problems as edge-corner boundary value problems | 514 |
| 8.4.2 | Operators near the smooth part of the crack boundary | 519 |
| 8.4.3 | Parameter-dependent crack operators on a sphere | 522 |
| 8.4.4 | The local corner-crack calculus | 526 |
| 8.4.5 | Singular crack problems | 530 |
| 8.4.6 | Examples | 532 |
| 8.4.7 | Further comments | 537 |
| 8.5 | Mixed problems with singular interfaces | 538 |
| 8.5.1 | Mixed problems in an infinite cylinder | 538 |
| 8.5.2 | Reduction to the boundary | 541 |
| 8.5.3 | Ellipticity with interface conditions | 545 |
| 9 | Operators in infinite cylinders and the relative index | 552 |
| 9.1 | Calculus with operator-valued meromorphic families | 552 |
| 9.1.1 | Characteristic values and factorisation of meromorphic families | 552 |
| 9.1.2 | The inhomogeneous equation | 559 |
| 9.1.3 | An index formula | 566 |
| 9.2 | Boundary value problems in infinite cylinders | 567 |
| 9.2.1 | Operators in cylindrical Sobolev spaces | 567 |
| 9.2.2 | Characteristic values and factorisation of meromorphic families | 571 |
| 9.2.3 | The relative index | 573 |
| 9.3 | The relative index for corner singularities | 576 |
| 9.3.1 | Parameter-dependent cone calculus | 576 |
| 9.3.2 | Meromorphic families | 581 |
| 9.3.3 | Examples | 587 |
| 9.3.4 | Characteristic values and factorisation | 590 |
| 9.3.5 | Operators on the infinite cylinder | 592 |
| 9.3.6 | The relative index | 597 |
| 9.4 | Cutting and pasting of elliptic operators | 597 |
| 9.4.1 | The locality of the index in the smooth case | 598 |
| 9.4.2 | Operators in bottleneck spaces | 599 |
| 9.4.3 | A general locality principle for the index | 602 |
| 10 | Intuitive ideas of the calculus on singular manifolds | 604 |
| 10.1 | Simple questions, unexpected answers | 604 |
| 10.1.1 | What is ellipticity? | 604 |
| 10.1.2 | Meromorphic symbolic structures | 615 |
| 10.1.3 | Naive and edge definitions of Sobolev spaces | 623 |
| 10.2 | Are regular boundaries harmless? | 630 |
| 10.2.1 | What is a boundary value problem? | 631 |
| 10.2.2 | Quantisation | 646 |
| 10.2.3 | The conormal cage | 656 |
| 10.3 | How interesting are conical singularities? | 661 |

| | | |
|--------|---|------------|
| 10.3.1 | The iterative construction of higher singularities | 662 |
| 10.3.2 | Operators with sleeping parameters | 666 |
| 10.3.3 | Smoothing operators may contribute to the index | 669 |
| 10.3.4 | Are cylinders the genuine corners? | 672 |
| 10.4 | Is ‘degenerate’ bad? | 674 |
| 10.4.1 | Operators on stretched spaces | 675 |
| 10.4.2 | What is ‘smoothness’ on a singular manifold? | 678 |
| 10.4.3 | Schwartz kernels and Green operators | 680 |
| 10.4.4 | Pseudo-differential aspects, solvability of equations | 682 |
| 10.4.5 | Discrete, branching, and continuous asymptotics | 686 |
| 10.5 | Higher generations of calculi | 694 |
| 10.5.1 | Higher generations of weighted corner spaces | 695 |
| 10.5.2 | Additional edge conditions in higher corner algebras | 699 |
| 10.5.3 | A hierarchy of topological obstructions | 701 |
| 10.5.4 | The building of singular algebras | 703 |
| 10.6 | Historical background and future program | 707 |
| 10.6.1 | Achievements of the past development | 707 |
| 10.6.2 | Conification and edgification | 710 |
| 10.6.3 | Similarities and differences between ellipticity and parabolicity | 714 |
| 10.6.4 | Open problems and new challenges | 719 |
| 10.6.5 | Concluding remarks | 723 |
| | Bibliography | 729 |
| | List of Symbols | 743 |
| | Index | 751 |