

Contents

Introduction	xiii
Chapter 1. Experimental and Numerical Analysis of Local Variables in Granular Materials	1
Farhang RADJAÏ and Jack LANIER	
1.1. Introduction	1
1.2. Description of granular texture	3
1.2.1. Particle connectivity	3
1.2.2. Contact network anisotropy: fabric tensors	5
1.2.2.1. General case	5
1.2.2.2. Case of 2D data	7
1.2.2.3. Case of 3D data	11
1.2.3. Branch vectors	13
1.2.4. Evolution of granular texture	14
1.2.5. Space partition: tessellation	17
1.2.5.1. Voronoi cells	17
1.2.5.2. Dirichlet cells	17
1.2.5.3. General case	18
1.2.5.4. Neighborhoods and local void ratios	18
1.3. Granular kinematics	20
1.3.1. Particle displacements and rotations	20
1.3.2. Rolling versus sliding	21
1.3.3. Fluctuating displacement fields	23
1.3.3.1. Uniform strain and fluctuations	23
1.3.3.2. Probability densities	25
1.3.3.3. Spatial correlations	26
1.3.3.4. Granulence	27
1.3.4. Local and global strains	27
1.3.4.1. Particle-scale strain	28

1.3.4.2. Strain localization	29
1.4. Force transmission	31
1.4.1. Probability density functions	32
1.4.2. Bimodal character of stress transmission	39
1.4.3. Force anisotropy	42
1.5. Conclusion	44
1.6. Bibliography	45

Chapter 2. The Stress Tensor in Granular Media and in other Mechanical Collections

51

Jean-Jacques MOREAU

2.1. Introduction	51
2.1.1. Motivation	52
2.1.2. The theoretical background	53
2.1.3. Dynamics	55
2.1.4. Pertinence	56
2.2. Efforts and virtual power	59
2.2.1. Resultant and moment of an effort	59
2.2.2. Internal efforts	60
2.2.3. Forces	62
2.2.4. Efforts of order greater than zero	62
2.2.5. Contact actions	64
2.3. Equilibrium	65
2.3.1. Main equalities	65
2.3.1.1. Case of a continuous body	65
2.3.1.2. Case of a granular material	66
2.3.2. Classical continuous body	67
2.3.3. Piece of string	68
2.3.4. Finite collection of points	71
2.3.5. Interaction bridges	73
2.3.6. Saturated soil	75
2.4. Comparison with the pair-by-pair approach	76
2.4.1. The classical definition	76
2.4.1.1. The pair-by-pair calculation	77
2.4.2. Numerical discussion of a tri-axial test	79
2.5. Directions of cut	83
2.5.1. Force transmitted across a cut	83
2.5.2. Proof of the cutting direction law	84
2.5.3. 2D bank	87
2.5.3.1. Free surface law	87
2.5.4. Conical pile	88
2.6. Coarse graining the equation of Statics	90
2.6.1. The divergence operator	90

2.7. One step into Dynamics	91
2.7.1. Introducing the acceleration field	91
2.7.2. Rigid bodies	92
2.7.2.1. Introducing the mass center	92
2.7.2.2. Introducing principal axes	93
2.7.2.3. Invoking rigid body dynamics	94
2.7.2.4. Spherical inertia	95
2.7.2.5. 2D models	95
2.7.3. Percussions	96
2.8. Bibliography	97
Chapter 3. Multiscale Techniques for Granular Materials	101
Bernard CAMBOU, Alexandre DANESCU	
3.1. Introduction	101
3.2. Scale change and fabric tensors	102
3.2.1. Solid particles description	102
3.2.1.1. Size of particles	102
3.2.1.2. Shape of particles	102
3.2.2. Fabric description for a granular sample	106
3.2.2.1. Coordination number and compactness	106
3.2.2.2. Definition of the overall anisotropy of a sample	108
3.2.3. Voids description	111
3.3. Change of scale for static variables	112
3.4. Change of scale for kinematic variables in granular materials	115
3.4.1. Definition of local kinematic variables	115
3.4.2. Method based on an energetic approach	117
3.4.3. Definition of strain from a discrete equivalent continuum	118
3.4.3.1. Strain proposed by Kruyt and Rothenburg in 2D	118
3.4.3.2. Strain proposed by Cambou <i>et al.</i> in 2D	121
3.4.3.3. Strain proposed by Bagi	123
3.4.4. Strain defined from best-fit methods	123
3.4.4.1. Strain proposed by Cundal	123
3.4.4.2. Strain proposed by Liao <i>et al.</i>	124
3.4.4.3. Strain proposed by Cambou <i>et al.</i>	126
3.4.5. Analysis of the different microstructural definitions of strain and comparison with the macro strain defined at the considered sample scale	126
3.5. Statistical homogenization in granular materials	131
3.5.1. Elastic behavior of a granular sample	132
3.5.1.1. Model based on kinematic localization	132
3.5.1.2. Model based on static localization	133
3.5.2. Elastic behavior of a granular sample	135
3.5.2.1. Voigt-type hypothesis for kinematic localization	135

3.5.2.2. Static localization hypothesis	136
3.5.3. Extension to nonlinear elasticity	139
3.5.4. Definition of a yield surface from a local criterion	140
3.5.4.1. Remarks on the validity of the yield criteria	143
3.5.5. Difficulties and limitations for statistical homogenization in granular materials	143
3.6. Bibliography	145
Chapter 4. Numerical Simulation of Granular Materials	149
Michel JEAN	
4.1. Introduction	149
4.2. The actors of a contact problem	152
4.2.1. Bodies, contactors and candidates to contact	153
4.2.2. Some bodies and contactors used in numerical simulation	156
4.2.3. Sorting	164
4.3. Kinematic relations	167
4.3.1. Usual rigid body kinematics	167
4.3.2. Local variables	168
4.3.3. The distance function	169
4.3.4. Relations between generalized and local variables	170
4.3.4.1. The mappings H and H^*	172
4.3.4.2. Non-uniqueness	172
4.3.5. Boundary conditions, driven or locked degrees of freedom	173
4.4. The dynamical equation	174
4.4.1. 2D or 3D bodies	175
4.4.2. Deformable bodies	176
4.4.3. Shocks, momentum, impulses and percussions	176
4.4.4. Energy formulae	178
4.5. Frictional contact laws	179
4.5.1. Unilaterality	180
4.5.1.1. Signorini conditions	180
4.5.1.2. Complementary relation and convex analysis	181
4.5.1.3. Flexibility models	182
4.5.1.4. Shock laws	184
4.5.2. Friction laws	186
4.5.2.1. Coulomb's law	186
4.5.2.2. Coulomb's law and convex analysis	188
4.5.2.3. Coulomb-type friction laws, strongly viscous at slow sliding speeds	189
4.5.2.4. Dynamical friction, static friction coefficients	189
4.5.3. Choosing a frictional contact law	191
4.5.4. Cohesive behavior	192
4.5.4.1. Mohr-Coulomb law	195

4.5.4.2. Rolling grains, welded grains	195
4.6. The equations governing a collection of contacting bodies	196
4.7. Preparing numerical samples	198
4.7.1. Boundary conditions	199
4.7.2. Initial state	203
4.7.3. Size of samples	205
4.8. Smooth DEM numerical methods	206
4.8.1. Molecular dynamics methods	206
4.8.2. Smooth DEM methods	207
4.8.2.1. Discretizing the dynamical equation	207
4.8.2.2. Discretizing the function $reac(q, u)$	208
4.8.3. PCDEM methods	208
4.8.3.1. Discrete form of the frictional law $Reac$	211
4.8.3.2. Proof of equation (4.40)	212
4.8.3.3. Numerical scheme	212
4.8.3.4. Remarks	213
4.8.3.5. Damping $C = aM$	214
4.8.4. Choosing the time-step	214
4.9. Non-smooth DEM numerical methods	216
4.9.1. Event-driven methods	216
4.9.1.1. Computing a collision	216
4.9.1.2. Remark	220
4.9.2. Non-smooth contact dynamics method	220
4.9.2.1. Discretization of the dynamical equation	220
4.9.2.2. Discrete form of kinematic relations	221
4.9.2.3. Discrete forms of frictional contact relations	222
4.9.2.4. Restriction of the dynamical equation to candidates to contact	223
4.9.2.5. Signorini μ -Coulomb standard problem	224
4.9.2.6. Remarks	224
4.9.2.7. Solving the Signorini μ -Coulomb standard problem	224
4.9.2.8. Solution of the 2D Signorini μ -Coulomb standard problem	226
4.9.2.9. Solving the frictional contact problem, Gauss Seidel nesting	226
4.10. Some illustrating examples	227
4.10.1. The bouncing ball problem	227
4.10.2. Frictional contact examples by explicit or implicit methods	230
4.10.2.1. Example 1	231
4.10.2.2. Example 2	231
4.10.2.3. Example 3	233
4.10.2.4. Example 4	233
4.11. Quasi-static evolutions, equilibrium dedicated methods	234
4.11.1. A strongly viscous contact law	235
4.11.2. Flexibility models	236
4.11.3. Rigid bodies and Signorini, μ -Coulomb law	237

4.11.4. Quasi-static evolutions versus dynamics	238
4.12. Accuracy criteria	241
4.12.1. Implicit methods	242
4.12.2. Explicit methods	243
4.12.3. Some accuracy estimators	243
4.12.3.1. Mean and quadratic violations	243
4.12.3.2. Bipotential violation	245
4.13. Indetermination in granular materials	246
4.13.1. A wedged disk example	248
4.13.1.1. A rigid wedged disk example	248
4.13.1.2. Analyzing the kinematic indetermination	249
4.13.1.3. A classical example of deformable model	250
4.13.1.4. Investigating indetermination, numerical experiments	253
4.13.1.5. The single wedged disk	253
4.13.1.6. Loading experiment, domains of attraction	255
4.13.1.7. Another view of domains of attraction: rigid model	257
4.13.1.8. Loading-unloading experiments	260
4.13.1.9. Indetermination in the deformable model	260
4.13.2. Three wedged disks, 200 packed disks examples	261
4.13.2.1. Three wedged disks	261
4.13.2.2. 200 disks sample	263
4.13.3. Conclusions	265
4.14. Stability	265
4.14.1. Perturbations	266
4.14.2. Coulomb stable sample	267
4.14.3. Left reactions perturbations of the single wedged disk	268
4.14.4. Left reactions perturbations of a 2,400 polygon sample	269
4.14.5. Further comments	274
4.15. Numerical integration schemes	275
4.15.1. θ method	276
4.15.2. Consistency of the discrete approximations	278
4.15.3. Newmark method	280
4.15.4. Deformable grains	281
4.15.5. Further comments	283
4.16. More non-smooth DEM methods	284
4.16.1. The NSCD method, Gauss–Seidel nesting	284
4.16.2. The NSCD method, Jacobi nesting	285
4.16.3. The bi-potential method	286
4.16.4. The generalized Newton method	287
4.16.5. Gradient-type methods	289
4.16.6. Mathematical programming methods	290
4.16.7. Multigrid computation	290
4.16.8. Parallel computation	291

4.17. Signorini μ -Coulomb derived laws	292
4.17.1. Status	293
4.17.1.1. Status SIGNORINI_CONTACT	293
4.17.1.2. Status COULOMB_SLIDING and COULOMB_STICKING	294
4.17.2. Change of variables	294
4.17.2.1. Remarks	296
4.17.3. Algorithm NSCD and derived laws	296
4.17.4. Gap Signorini condition and Coulomb's law	297
4.17.5. Inelastic quasi-choc law and Coulomb's law	297
4.17.6. Velocity Signorini condition and Coulomb's law	298
4.17.7. Restitution shock law together with Coulomb's law	298
4.17.8. Velocity Signorini condition and Coulomb's law with static or dynamic friction coefficient	298
4.17.9. Flexible unilateral conditions and Coulomb's law	299
4.17.10. Mohr Coulomb cohesive law	299
4.17.11. A simple cohesive example	300
4.17.12. Fiber-like materials	301
4.18. Conclusion	301
4.19. Appendix: basic convex analysis	303
4.19.1. Convex sets and cones	303
4.19.2. Convex functions, conjugates, subdifferential	303
4.19.3. Standard Signorini relation	305
4.19.4. Standard Coulomb's law	305
4.19.5. Bipotentials	306
4.20. Bibliography	307

Chapter 5. Frictionless Unilateral Multibody Dynamics 317

Patrick BALLARD

5.1. Introduction	317
5.2. The dynamics of rigid body systems	318
5.2.1. The geometric description	318
5.2.2. Formulation of the dynamics	319
5.2.3. Well-posedness of the dynamics	321
5.3. The dynamics of rigid body systems with perfect bilateral constraints	322
5.3.1. The geometric description	322
5.3.2. Formulation of the dynamics	323
5.3.3. Well-posedness of the dynamics	325
5.4. The dynamics of rigid body systems with perfect unilateral constraints	326
5.4.1. The geometric description	326
5.4.2. Formulation of the dynamics	327
5.4.2.1. Equation of motion	327
5.4.2.2. The impact constitutive equation	329
5.4.2.3. The evolution problem	333

5.4.3. Well-posedness of the dynamics	334
5.4.3.1. Comments	339
5.5. Bibliography	340
List of Authors	343
Index	345