Contents

1	For	mulations	1
	1.1	Modeling techniques	2
	1.2	Guidelines for strong formulations	8
	1.3	Modeling with exponentially many constraints	13
	1.4	Modeling with exponentially many variables	23
	1.5	Summary	25
	1.6	Exercises	25
	1.7	Notes and sources	33
2	Me	thods to enhance formulations	35
	2.1	Methods to generate valid inequalities	36
	2.2	Methods to generate facet defining inequalities	46
	2.3	Valid inequalities in independence systems	52
	2.4	On the strength of valid inequalities	56
	2.5	Nonlinear formulations	61
	2.6	Summary	69
	2.7	Exercises	69
	2.8	Notes and sources	76
3	Idea	al formulations	79
	3.1	Total unimodularity	80
	3.2	Dual methods	87
	3.3	Randomized rounding methods	100
	3.4	The minimal counterexample method	109
	3.5	The method of lift and project	111
	3.6	Summary	116
	3.7	Exercises	117
	3.8	Notes and sources	120
4	Dua	ality in integer optimization	123
	4.1	Duality of binary optimization	124
	4.2	Superadditive duality	134
	4.3	Lagrangean duality	141
	4.4	Geometry and solution of Lagrangean duals	149

Contents	;
----------	---

	4.5	Summary	159
	4.6	Exercises	160
	4.7	Notes and sources	164
5	Alg	orithms for solving relaxations	165
	5.1	The key geometric result behind the ellipsoid method	166
	5.2	The ellipsoid method for the feasibility problem	173
	5.3	The ellipsoid method for optimization	181
	5.4	From separation to optimization	183
	5.5	Summary	191
	5.6	Exercises	191
	5.7	Notes and sources	196
6	Lat	tices and their applications	199
	6.1	Integer points in lattices	200
	6.2	Reduced bases for lattices	211
	6.3	Simultaneous diophantine approximation	228
	6.4	The approximate nearest vector problem	230
	6.5	The maximum volume inscribed ellipsoid	233
	6.6	Integer optimization in fixed dimension	235
	6.7	Summary	242
	6.8	Exercises	242
	6.9	Notes and sources	245
7	Alg	ebraic geometry and integer optimization	247
	7.1	Background from algebraic geometry	248
	7.2	Applications to binary optimization	259
	7.3	Gröbner bases for integer optimization	264
	7.4	Applications of real algebraic geometry	269
	7.5	Generating functions for integer points in polyhedra	274
	7.6	Summary	281
	7.7	Exercises	282
	7.8	Notes and sources	284
8	Geo	ometry of integer optimization	285
	8.1	Definitions and examples	286
	8.2	Integral generating sets in cones	289
	8.3	Optimality conditions	295
	8.4	From cones to polyhedra	298
	8.5	Algorithms to compute integral generating sets	305
	8.6	Total dual integrality	312
	8.7	Summary	320
	8.8	Exercises	321
	8.9	Notes and sources	323

9	Cut	ting plane methods	325
	9.1	Cutting planes from integral generating sets	326
	9.2	The Gomory cutting plane algorithm	333
	9.3	Cutting planes based on lattices	335
	9.4	The convex hull of solutions	341
	9.5	Summary	349
	9.6	Exercises	349
	9.7	Notes and sources	352
10	The	internal basis method	255
10	10.1	Duranie reformulation methods	256
	10.1	An integer simpley algorithm	262
	10.2	The integer simplex algorithm	303
	10.3	Algebraic reference et	300
	10.4	Algebraic reformulation strategies	010
	10.0	Combinatorial reformulation strategies	303
	10.0	Summary	391
	10.7	Exercises	391
	10.8	Notes and sources	393
11	Enu	merative and heuristic methods	395
	11.1	Branch and bound	396
	11.2	Optimization based heuristic methods	406
	11.3	Dynamic programming	408
	11.4	Approximate dynamic programming	414
	11.5	Local search	416
	11.6	Simulated annealing	418
	11.7	Summary	420
	11.8	Exercises	421
	11.9	Notes and sources	425
12	App	roximation algorithms	427
	12.1	Primal-dual methods	429
	12.2	Cut covering problems	440
	12.3	Randomized rounding of linear relaxations	447
	12.4	Randomized rounding of convex relaxations	461
	12.5	Approximation schemes	469
	12.6	Limitations in approximability	475
	12.7	Summary	476
	12.8	Exercises	478
	12.9	Notes and sources $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	484
13	Mix	ed integer optimization	487
	13.1	The mixed integer Farkas lemma	488
	13.2	Mixed integer lattices	494
	13.2	Mixed integer optimality conditions	496
	12/	Reformulations for mixed integer sats	500
	10.4	recommutations for mixed integer sets	000

,

	13.5	Cutting planes for mixed integer optimization	505
	13.6	Summary	520
	13.7	Exercises	520
	13.8	Notes and Sources	522
14	Rob	oust discrete optimization	525
	14.1	Robust mixed integer optimization	526
	14.2	Robust binary optimization	532
	14.3	Robust network flows	536
	14.4	Robust inventory theory	540
	14.5	Summary	546
	14.6	Exercises	546
	14.7	Notes and sources	549
A	Eler	nents of polyhedral theory	551
A	Eler A.1	nents of polyhedral theory Cones	551 552
A	Eler A.1 A.2	nents of polyhedral theory Cones	551 552 553
A	Eler A.1 A.2 A.3	nents of polyhedral theory Cones	551 552 553 555
A	Eler A.1 A.2 A.3 A.4	nents of polyhedral theory Cones Dimension of polyhedra Valid inequalities Exercises	551 552 553 555 558
A	Eler A.1 A.2 A.3 A.4 Effic	nents of polyhedral theory Cones Dimension of polyhedra Valid inequalities Exercises cient algorithms and complexity theory	551 552 553 555 558 559
A B	Eler A.1 A.2 A.3 A.4 Effic B.1	nents of polyhedral theory Cones Dimension of polyhedra Valid inequalities Exercises cient algorithms and complexity theory Efficient algorithms	551 552 553 555 558 559 560
A B	Eler A.1 A.2 A.3 A.4 Effic B.1 B.2	nents of polyhedral theory Cones Dimension of polyhedra Valid inequalities Exercises cient algorithms and complexity theory Efficient algorithms Complexity theory	551 552 553 555 558 559 560 563
A B Re	Eler A.1 A.2 A.3 A.4 Effic B.1 B.2	nents of polyhedral theory Cones Dimension of polyhedra Valid inequalities Exercises cient algorithms and complexity theory Efficient algorithms Complexity theory Complexity theory	 551 552 553 555 558 559 560 563 571