

# Contents

Preface	vii
Chapter 1. Preliminaries	1
1.1. The Hardy space	2
1.2. The Dirichlet space	3
1.3. Tree spaces	5
Chapter 2. The Interpolation Problem	9
2.1. Origins of interpolation in the corona problem	9
2.2. Origins of interpolation in control theory	10
2.3. Carleson's duality proof of interpolation	12
2.4. Peter Jones' constructive proof of interpolation	23
2.5. Other interpolation problems	32
Chapter 3. The Corona Problem	35
3.1. Commutative Banach algebras	35
3.2. Wolff's proof of Carleson's Corona Theorem	40
3.3. The corona $\bar{d}$ equation	43
3.4. Other corona problems	45
Chapter 4. Toeplitz and Hankel Operators	55
4.1. $H^1$ - $BMO$ duality	58
4.2. Compact Hankel operators	64
4.3. Best approximation	69
Chapter 5. Hilbert Function Spaces and Nevanlinna-Pick Kernels	87
5.1. The commutant	87
5.2. Higher dimensions	102
5.3. Applications of Carleson measures	107
5.4. Interpolating sequences for certain spaces with NP kernel	112
5.5. The corona problem for multiplier spaces in $\mathbb{C}^n$	118
Chapter 6. Carleson Measures for the Hardy-Sobolev Spaces	133
6.1. Unified proofs for trees	135
6.2. Invariant metrics, measures and derivatives	137
6.3. Carleson measures on the ball $\mathbb{B}_n$	140
Appendix A. Functional Analysis	145
A.1. Banach spaces and bounded linear operators	148
A.2. Hilbert spaces	149

A.3. Duality	152
A.4. Completeness theorems	155
A.5. Convexity theorems	162
A.6. Compact operators	168
Appendix B. Weak Derivatives and Sobolev Spaces	173
B.1. Weak derivatives	173
B.2. The Sobolev space $W^{1,2}(\Omega)$	174
B.3. Maximal functions	177
B.4. Bounded mean oscillation and the John-Nirenberg inequality	180
Appendix C. Function Theory on the Disk	183
C.1. Factorization theorems	188
C.2. The shift operator	191
Appendix D. Spectral Theory for Normal Operators	195
D.1. Positive operators	197
Bibliography	199
Index	201