

# Contents

<b>Preface</b>	<b>ix</b>
<b>1 Introduction</b>	<b>1</b>
1.1 The Moonshine Theorem . . . . .	2
1.1.1 A Brief History . . . . .	2
1.1.2 The Theorem . . . . .	3
1.2 Borcherds-Kac-Moody Lie Superalgebras . . . . .	5
1.3 Vector Valued Modular Forms . . . . .	7
1.4 Borcherds-Kac-Moody Lie Algebras and Modular forms . . . . .	7
1.5 $\Gamma$ -graded Vertex Algebras . . . . .	9
1.6 A Construction of a Class of Borcherds-Kac-Moody Lie (super)algebras . . . . .	9
<b>2 Borcherds-Kac-Moody Lie Superalgebras</b>	<b>13</b>
2.1 Definitions and Elementary Properties . . . . .	13
2.2 Bilinear Forms . . . . .	26
2.3 The Root System . . . . .	38
2.4 Uniqueness of the Generalized Cartan Matrix . . . . .	58
2.5 A Characterization of BKM Superalgebras . . . . .	66
2.6 Character and Denominator Formulas . . . . .	72
<b>3 Singular Theta Transforms of Vector Valued Modular Forms</b>	<b>93</b>
3.1 Lattices . . . . .	93
3.2 Ordinary Modular Functions . . . . .	95
3.3 Vector Valued Modular Functions . . . . .	102
3.4 The Singular Theta Correspondence . . . . .	113
<b>4 <math>\Gamma</math>-Graded Vertex Algebras</b>	<b>129</b>
4.1 The Structure of $\Gamma$ -graded Vertex Algebras . . . . .	129
4.2 $\Gamma$ -Graded Lattice Vertex Algebras . . . . .	149
4.3 From Lattice Vertex Algebras to Lie Algebras . . . . .	167

<b>5 Lorentzian BKM Algebras</b>	<b>177</b>
5.1 Introduction . . . . .	177
5.2 Automorphic forms on Grassmannians . . . . .	179
5.3 Vector Valued Modular forms and LBKM Algebras . . . . .	186
5.4 An Upper Bound for the Rank of the Root Lattices of LBKM Algebras? . . . . .	207
5.5 A Construction of LBKM Algebras from Lattice Vertex Algebras . . . . .	213
<b>A Orientations and Isometry Groups</b>	<b>239</b>
<b>B Manifolds</b>	<b>241</b>
B.1 Some Elementary Topology . . . . .	241
B.2 Manifolds . . . . .	242
B.3 Fibre Bundles and Covering Spaces . . . . .	247
<b>C Some Complex Analysis</b>	<b>251</b>
C.1 Measures and Lebesgue Integrals . . . . .	251
C.2 Complex Functions . . . . .	253
C.3 Integration . . . . .	254
C.4 Some Special Functions . . . . .	255
<b>D Fourier Series and Transforms</b>	<b>263</b>
<b>Index</b>	<b>283</b>