

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Basic Concepts and Definitions</b>	<b>11</b>
2.1	Autonomous Dynamical Systems . . . . .	12
2.1.1	Hyperbolic Sets and Invariant Manifolds . . . . .	13
2.1.2	Attractor . . . . .	14
2.1.3	Ergodic Theory . . . . .	16
2.1.4	Lyapunov Exponents . . . . .	16
2.2	Non-Autonomous Dynamical Systems . . . . .	20
2.2.1	Hyperbolic Trajectories and Invariant Manifolds . . . . .	20
2.2.2	Pullback Attractor . . . . .	25
2.2.3	Lyapunov Exponents . . . . .	26
2.3	Finite-Time Velocity Fields . . . . .	27
<b>3</b>	<b>The Set Oriented Approach</b>	<b>30</b>
3.1	The Subdivision Algorithm . . . . .	30
3.2	The Continuation Algorithm . . . . .	32
3.3	Extensions to Non-Autonomous Systems . . . . .	34
<b>4</b>	<b>Expansion Rate Approach for Autonomous Systems</b>	<b>37</b>
4.1	Introduction . . . . .	37
4.2	Theoretical Results . . . . .	39
4.2.1	Expansion Rates . . . . .	39
4.2.2	Expansion Rates and Stable Manifolds . . . . .	39
4.2.3	Direct Expansion Rates . . . . .	43
4.2.4	Error Estimates . . . . .	44
4.3	Approximation of Expansion Rates . . . . .	47
4.3.1	Set Wise Expansion Rates . . . . .	47
4.3.2	Test Point Strategies . . . . .	47
4.3.3	Choice of Number of Iterations $N$ . . . . .	50
4.3.4	Parallel Computing . . . . .	53

4.3.5	Examples . . . . .	54
4.4	Extraction of Invariant Manifolds . . . . .	60
4.4.1	The Subdivision Algorithm . . . . .	60
4.4.2	Examples . . . . .	62
4.5	Expansion in Graphs . . . . .	68
4.5.1	Dynamical Systems and Graphs . . . . .	68
4.5.2	Graph Based Expansion . . . . .	69
4.5.3	Examples . . . . .	70
4.5.4	Discussion . . . . .	73
<b>5</b>	<b>Analyzing Transport in Non-Autonomous Systems</b>	<b>76</b>
5.1	Introduction . . . . .	76
5.2	Time-Dependent Invariant Manifolds . . . . .	77
5.2.1	Theoretical Results . . . . .	79
5.2.2	Numerical Methods . . . . .	83
5.2.3	The Continuation Algorithm . . . . .	83
5.2.4	Examples . . . . .	85
5.3	Computation of Transport Rates . . . . .	92
5.3.1	A Transfer Operator Approach . . . . .	93
5.3.2	Examples . . . . .	100
<b>6</b>	<b>Application to Ocean Flows</b>	<b>108</b>
6.1	Introduction . . . . .	108
6.2	Two-Dimensional Ocean Flows . . . . .	110
6.3	Transport in Monterey Bay . . . . .	115
6.4	Three-Dimensional Ocean Flows . . . . .	116
<b>7</b>	<b>Conclusion</b>	<b>120</b>