

CONTENTS

Lecture L1: QUADRATIC EQUATIONS	1
§1: Word Problems	1
§2: Sets and Maps	3
§3: Groups and Fields	3
§4: Rings and Ideals	6
§5: Modules and Vector Spaces	8
§6: Polynomials and Rational Functions	9
§7: Euclidean Domains and Principal Ideal Domains	13
§8: Root Fields and Splitting Fields	14
§9: Advice to the Reader	16
§10: Definitions and Remarks	16
§11: Examples and Exercises	23
§12: Notes	27
§13: Concluding Note	29
Lecture L2: CURVES AND SURFACES	30
§1: Multivariable Word Problems	30
§2: Power Series and Meromorphic Series	34
§3: Valuations	39
§4: Advice to the Reader	43
§5: Zorn's Lemma and Well Ordering	44
§6: Utilitarian Summary	52
§7: Definitions and Exercises	52
§8: Notes	59
§9: Concluding Note	60
Lecture L3: TANGENTS AND POLARS	61
§1: Simple Groups	61
§2: Quadrics	63
§3: Hypersurfaces	64
§4: Homogeneous Coordinates	66

§5: Singularities	70
§6: Hensel's Lemma and Newton's Theorem	72
§7: Integral Dependence	77
§8: Unique Factorization Domains	81
§9: Remarks	82
§10: Advice to the Reader	83
§11: Hensel and Weierstrass	83
§12: Definitions and Exercises	90
§13: Notes	98
§14: Concluding Note	98
Lecture L4: VARIETIES AND MODELS	100
§1: Resultants and Discriminants	100
§2: Varieties	104
§3: Noetherian Rings	105
§4: Advice to the Reader	107
§5: Ideals and Modules	108
§6: Primary Decomposition	134
§6.1: Primary Decomposition for Modules	136
§7: Localization	137
§7.1: Localization at a Prime Ideal	144
§8: Affine Varieties	146
§8.1: Spectral Affine Space	152
§8.2: Modelic Spec and Modelic Affine Space	152
§8.3: Simple Points and Regular Local Rings	153
§9: Models	154
§9.1: Modelic Proj and Modelic Projective Space	157
§9.2: Modelic Blowup	159
§9.3: Blowup of Singularities	160
§10: Examples and Exercises	161
§11: Problems	171
§12: Remarks	172
§13: Definitions and Exercises	195
§14: Notes	200
§15: Concluding Note	201
Lecture L5: PROJECTIVE VARIETIES	202
§1: Direct Sums of Modules	202
§2: Graded Rings and Homogeneous Ideals	206
§3: Ideal Theory in Graded Rings	209
§4: Advice to the Reader	216
§5: More about Ideals and Modules	216

(Q1) Nilpotents and Zerodivisors in Noetherian Rings	216
(Q2) Faithful Modules and Noetherian Conditions	218
(Q3) Jacobson Radical, Zariski Ring, and Nakayama Lemma	219
(Q4) Krull Intersection Theorem and Artin-Rees Lemma	220
(Q5) Nagata's Principle of Idealization	225
(Q6) Cohen's and Eakin's Noetherian Theorems	229
(Q7) Principal Ideal Theorems	230
(Q8) Relative Independence and Analytic Independence	236
(Q9) Going Up and Going Down Theorems	241
(Q10) Normalization Theorem and Regular Polynomials	247
(Q11) Nilradical, Jacobson Spectrum, and Jacobson Ring	261
(Q12) Catenarian Rings and Dimension Formula	268
(Q13) Associated Graded Rings and Leading Ideals	272
(Q14) Completely Normal Domains	277
(Q15) Regular Sequences and Cohen-Macaulay Rings	280
(Q16) Complete Intersections and Gorenstein Rings	300
(Q17) Projective Resolutions of Finite Modules	311
(Q18) Direct Sums of Algebras, Reduced Rings, and PIRs	340
(Q18.1) Orthogonal Idempotents and Ideals in a Direct Sum	341
(Q18.2) Localizations of Direct Sums	344
(Q18.3) Comaximal Ideals and Ideal Theoretic Direct Sums	345
(Q18.4) SPIRs = Special Principal Ideal Rings	348
(Q19) Invertible Ideals, Conditions for Normality, and DVRs	354
(Q20) Dedekind Domains and Chinese Remainder Theorem	364
(Q21) Real Ranks of Valuations and Segment Completions	372
(Q22) Specializations and Compositions of Valuations	381
(Q23) UFD Property of Regular Local Domains	385
(Q24) Graded Modules and Hilbert Polynomials	393
(Q25) Hilbert Polynomial of a Hypersurfaces	397
(Q26) Homogeneous Submodules of Graded Modules	399
(Q27) Homogeneous Normalization	401
(Q28) Alternating Sum of Lengths	408
(Q29) Linear Disjointness and Intersection of Varieties	414
(Q30) Syzygies and Homogeneous Resolutions	433
(Q31) Projective Modules Over Polynomial Rings	441
(Q32) Separable Extensions and Primitive Elements	514
(Q33) Restricted Domains and Projective Normalization	529
(Q34) Basic Projective Algebraic Geometry	534
(Q34.1) Projective Spectrum	534
(Q34.2) Homogeneous Localization	536
(Q34.3) Varieties in Projective Space	541
(Q34.4) Projective Decomposition of Ideals and Varieties	545

(Q34.5) Modelic and Spectral Projective Spaces	547
(Q34.6) Relation between Affine and Projective Varieties	548
(Q35) Simplifying Singularities by Blowups	552
(Q35.1) Hypersurface Singularities	552
(Q35.2) Blowing-up Primary Ideals	553
(Q35.3) Residual Properties and Coefficient Sets	555
(Q35.4) Geometrically Blowing-up Simple Centers	555
(Q35.5) Algebraically Blowing-up Simple Centers	559
(Q35.6) Dominating Modelic Blowup	566
(Q35.7) Normal Crossings, Equimultiple Locus, and Simple Points	567
(Q35.8) Quadratic and Monoidal Transformations	569
(Q35.9) Regular Local Rings	577
§6: Definitions and Exercises	578
§7: Notes	596
§8: Concluding Note	597
Lecture L6: PAUSE AND REFRESH	598
§1: Summary of Lecture L1 on Quadratic Equations	598
§2: Summary of Lecture L2 on Curves and Surfaces	603
§3: Summary of Lecture L3 on Tangents and Polars	606
§4: Summary of Lecture L4 on Varieties and Models	608
§5: Summary of Lecture L5 on Projective Varieties	611
§6: Definitions and Exercises	634
BIBLIOGRAPHY	689
DETAILED CONTENT	691
NOTATION-SYMBOLS	713
NOTATION-WORDS	717
INDEX	725