Contents

1 Introduction 1

1.1 Walkthrough systems . . . 2

1.2 Geometric spanners and weak spanners . . . 3

1.3 Fault tolerant spanners . . . 5

1.4 Computational model . . . 6

1.5 Outline . . . 7

2 The θ-graph 9

2.1 The two-dimensional θ-graph . . . 9

2.1.1 The spanner property . . . 10

2.1.2 Computation . . . 12

2.1.3 The weak spanner property for $\theta \leq \frac{\pi}{3}$ . . . 15

2.1.4 The weak spanner property for $\theta = \frac{\pi}{2}$ . . . 16

2.1.5 The reversed θ-graph . . . 21

2.1.6 Generalizations of $G_{\pi/2}(S)$ . . . 22

2.1.7 The graph $G_{2\pi/3}(S)$ . . . 29

2.2 The higher dimensional θ-graph . . . 31

2.2.1 The spanner property . . . 33

2.2.2 Construction of a θ-frame . . . 34

2.2.3 Construction of the θ-graph . . . 44

2.3 Conclusions and open problems . . . 48

3 Fault tolerant spanners 51

3.1 A k-vertex fault tolerant f-spanner with $O(kn)$ edges . . . 53

3.1.1 The ith order θ-graph . . . 53

3.1.2 The vertex fault tolerant spanner property . . . 54

3.1.3 Computation 55
3.2 k-edge fault tolerant f-spanners 57
3.3 A k-vertex fault tolerant f-spanner with degree $O(k^2)$ 58
3.3.1 k-vertex fault tolerant single sink spanners 58
3.3.2 A bounded degree k-vetrex fault tolerant f-spanner 61
3.4 Fault tolerant spanners with Steiner points 62
3.4.1 Upper bounds 62
3.4.2 Lower bounds 63
3.5 Conclusion and open problems 65

4 Applications in walkthrough systems **67**
4.1 Geometric search problems in walkthrough systems 67
4.1.1 Related problems, state of the art 69
4.1.2 Our approach 71
4.1.3 Outline 71
4.2 A randomized solution 72
4.2.1 The static data structure 72
4.2.2 The fully dynamic data structure 74
4.3 A deterministic solution 74
4.3.1 The static data structure 75
4.3.2 The incremental data structure, lazy updates 80
4.4 Objects with different sizes 81
4.4.1 Using circular range searching in size classes 82
4.4.2 Transformation to halfspace range searching 83
4.5 Conclusion and open problems 85

5 Lower bound on the construction time of weak spanners with Steiner points in the algebraic model **87**

Bibliography **93**