

Contents

1	Introduction	9
2	The Subdivision Algorithm	15
2.1	Introduction	15
2.2	The Algorithm	16
2.3	Realization	20
2.4	Estimates of the computational effort	22
3	Location of Zeros A: $g : \mathbb{R}^n \rightarrow \mathbb{R}^n$	27
3.1	Introduction	27
3.2	The Algorithms	28
3.3	Numerical Results	34
3.4	An "Application": Scalar Optimization	40
3.5	Conclusion	44
4	Location of Zeros B: $g : \mathbb{C} \rightarrow \mathbb{C}$	45
4.1	Introduction	45
4.2	Theoretical Background	46
4.3	The Algorithm	46
4.3.1	Description of a Basic Subdivision Scheme	47
4.3.2	Adaptive Version of the Basic Subdivision Scheme: the QZ-40 Algorithm	48
4.4	Numerical Results	50
4.4.1	Academic Example	50
4.4.2	Stability of a Ring Oscillator	50
4.4.3	Stability of an Annular Combustion Chamber	53
4.5	Conclusion	55
5	Computing the Stability Regions of Delay Differential Equations	57
5.1	Introduction	57
5.2	Theoretical Background	58
5.3	The Algorithm	59
5.4	Numerical Results	61
5.4.1	Example A	62
5.4.2	Example B	62

5.4.3	Example C	62
5.5	Conclusion	63
6	Multi-Objective Optimization	67
6.1	Introduction	67
6.2	Theoretical Background	69
6.2.1	Pareto Optimality	69
6.2.2	Convergence toward Pareto Sets	74
6.3	Basic Algorithms	77
6.3.1	Subdivision Algorithm	77
6.3.2	Recovering Algorithm	79
6.3.3	Sampling Algorithm	81
6.3.4	Usage and Combination of the Algorithms	82
6.4	Numerical Results for General Models	83
6.4.1	Example G1	83
6.4.2	Example G2	84
6.4.3	Example G3	84
6.4.4	Example G4	85
6.4.5	Example G5 – Optimization of an Active Suspension	87
6.5	A Data Structure for the Computation of the Nondominance Problem 91	
6.5.1	Introduction and Background	93
6.5.2	Attacking the Nondominance Problem	94
6.5.3	Computational Results	97
6.6	Extensions for Non-Smooth Models	98
6.6.1	Introduction	98
6.6.2	A Short Introduction to MOEA’s	98
6.6.3	The Algorithms	104
6.6.4	Using Archives	108
6.7	Numerical Results for Non-Smooth Models	112
6.7.1	Example N1	112
6.7.2	Example N2	113
6.8	Extensions for Smooth Models	113
6.8.1	Introduction	113
6.8.2	The Algorithms	114
6.8.3	Uniform Distribution of Solutions	117
6.9	Numerical Results for Smooth Models	119
6.9.1	Example S1	119
6.9.2	Example S2	119
6.9.3	Example S3	120
6.9.4	Example S4	120
6.9.5	Example S5	121
6.10	Conclusion and Future Work	121
7	Conclusion	135