

Contents

Preface	ix
Acknowledgments	xv
Part 1. Functions of One Variable	
Chapter 1. Monotone Functions	3
§1.1. Continuity	3
§1.2. Differentiability	8
Chapter 2. Functions of Bounded Pointwise Variation	39
§2.1. Pointwise Variation	39
§2.2. Composition in $BPV(I)$	55
§2.3. The Space $BPV(I)$	59
§2.4. Banach Indicatrix	66
Chapter 3. Absolutely Continuous Functions	73
§3.1. $AC(I)$ Versus $BPV(I)$	73
§3.2. Chain Rule and Change of Variables	94
§3.3. Singular Functions	107
Chapter 4. Curves	115
§4.1. Rectifiable Curves and Arclength	115
§4.2. Fréchet Curves	130
§4.3. Curves and Hausdorff Measure	134
§4.4. Jordan's Curve Theorem	146

Chapter 5. Lebesgue–Stieltjes Measures	155
§5.1. Radon Measures Versus Increasing Functions	155
§5.2. Signed Borel Measures Versus $BPV(I)$	161
§5.3. Decomposition of Measures	166
§5.4. Integration by Parts and Change of Variables	181
Chapter 6. Decreasing Rearrangement	187
§6.1. Definition and First Properties	187
§6.2. Absolute Continuity of u^*	202
§6.3. Derivative of u^*	209
Chapter 7. Functions of Bounded Variation and Sobolev Functions	215
§7.1. $BV(\Omega)$ Versus $BPV(\Omega)$	215
§7.2. Sobolev Functions Versus Absolutely Continuous Functions	222
Part 2. Functions of Several Variables	
Chapter 8. Absolutely Continuous Functions and Change of Variables	231
§8.1. The Euclidean Space \mathbb{R}^N	231
§8.2. Absolutely Continuous Functions of Several Variables	234
§8.3. Change of Variables for Multiple Integrals	242
Chapter 9. Distributions	255
§9.1. The Spaces $\mathcal{D}_K(\Omega)$, $\mathcal{D}(\Omega)$, and $\mathcal{D}'(\Omega)$	255
§9.2. Order of a Distribution	264
§9.3. Derivatives of Distributions and Distributions as Derivatives	266
§9.4. Convolutions	275
Chapter 10. Sobolev Spaces	279
§10.1. Definition and Main Properties	279
§10.2. Density of Smooth Functions	283
§10.3. Absolute Continuity on Lines	293
§10.4. Duals and Weak Convergence	298
§10.5. A Characterization of $W^{1,p}(\Omega)$	305
Chapter 11. Sobolev Spaces: Embeddings	311
§11.1. Embeddings: $1 \leq p < N$	312
§11.2. Embeddings: $p = N$	328
§11.3. Embeddings: $p > N$	335

§11.4. Lipschitz Functions	341
Chapter 12. Sobolev Spaces: Further Properties	349
§12.1. Extension Domains	349
§12.2. Poincaré Inequalities	359
Chapter 13. Functions of Bounded Variation	377
§13.1. Definition and Main Properties	377
§13.2. Approximation by Smooth Functions	380
§13.3. Bounded Pointwise Variation on Lines	386
§13.4. Coarea Formula for BV Functions	397
§13.5. Embeddings and Isoperimetric Inequalities	401
§13.6. Density of Smooth Sets	408
§13.7. A Characterization of $BV(\Omega)$	413
Chapter 14. Besov Spaces	415
§14.1. Besov Spaces $B^{s,p,\theta}$, $0 < s < 1$	415
§14.2. Dependence of $B^{s,p,\theta}$ on s	419
§14.3. The Limit of $B^{s,p,\theta}$ as $s \rightarrow 0^+$ and $s \rightarrow 1^-$	421
§14.4. Dependence of $B^{s,p,\theta}$ on θ	425
§14.5. Dependence of $B^{s,p,\theta}$ on s and p	429
§14.6. Embedding of $B^{s,p,\theta}$ into L^q	437
§14.7. Embedding of $W^{1,p}$ into $B^{t,q}$	442
§14.8. Besov Spaces and Fractional Sobolev Spaces	448
Chapter 15. Sobolev Spaces: Traces	451
§15.1. Traces of Functions in $W^{1,1}(\Omega)$	451
§15.2. Traces of Functions in $BV(\Omega)$	464
§15.3. Traces of Functions in $W^{1,p}(\Omega)$, $p > 1$	465
§15.4. A Characterization of $W_0^{1,p}(\Omega)$ in Terms of Traces	475
Chapter 16. Sobolev Spaces: Symmetrization	477
§16.1. Symmetrization in L^p Spaces	477
§16.2. Symmetrization of Lipschitz Functions	482
§16.3. Symmetrization of Piecewise Affine Functions	484
§16.4. Symmetrization in $W^{1,p}$ and BV	487
Appendix A. Functional Analysis	493
§A.1. Metric Spaces	493

§A.2. Topological Spaces	494
§A.3. Topological Vector Spaces	497
§A.4. Normed Spaces	501
§A.5. Weak Topologies	503
§A.6. Hilbert Spaces	506
 Appendix B. Measures	507
§B.1. Outer Measures and Measures	507
§B.2. Measurable and Integrable Functions	511
§B.3. Integrals Depending on a Parameter	519
§B.4. Product Spaces	520
§B.5. Radon–Nikodym’s and Lebesgue’s Decomposition Theorems	522
§B.6. Signed Measures	523
§B.7. L^p Spaces	526
§B.8. Modes of Convergence	534
§B.9. Radon Measures	536
§B.10. Covering Theorems in \mathbb{R}^N	538
 Appendix C. The Lebesgue and Hausdorff Measures	543
§C.1. The Lebesgue Measure	543
§C.2. The Brunn–Minkowski Inequality and Its Applications	545
§C.3. Convolutions	550
§C.4. Mollifiers	552
§C.5. Differentiable Functions on Arbitrary Sets	560
§C.6. Maximal Functions	564
§C.7. Anisotropic L^p Spaces	568
§C.8. Hausdorff Measures	572
 Appendix D. Notes	581
Appendix E. Notation and List of Symbols	587
Bibliography	593
Index	603