

# Table of Contents

<b>1.</b>	<b>The Foundations of Modern Cryptography</b>	1
1.1	Introduction	1
1.2	Central Paradigms	5
1.2.1	Computational Difficulty	7
1.2.2	Computational Indistinguishability	8
1.2.3	The Simulation Paradigm	8
1.3	Pseudorandomness	9
1.3.1	The Basics	9
1.3.2	Pseudorandom Functions	10
1.4	Zero-Knowledge	12
1.4.1	The Basics	12
1.4.2	Some Variants	13
1.5	Encryption	15
1.5.1	Definitions	15
1.5.2	Constructions	17
1.5.3	Security Beyond Passive Attacks	19
1.6	Signatures	20
1.6.1	Definitions	21
1.6.2	Constructions	21
1.6.3	Two Variants	23
1.7	Cryptographic Protocols	24
1.7.1	Definitions	25
1.7.2	Constructions	26
1.8	Some Notes	26
1.8.1	General Notes	27
1.8.2	Specific Notes	31
1.9	Historical Perspective	33
1.10	Two Suggestions for Future Research	35
1.11	Some Suggestions for Further Reading	36
<b>2.</b>	<b>Probabilistic Proof Systems</b>	39
2.1	Introduction	39
2.2	Interactive Proof Systems	41
2.2.1	Definition	41

2.2.2	The Role of Randomness .....	42
2.2.3	The Power of Interactive Proofs .....	43
2.2.4	The Interactive Proof System Hierarchy .....	47
2.2.5	How Powerful Should the Prover Be? .....	48
2.3	Zero-Knowledge Proof Systems .....	49
2.3.1	A Sample Definition .....	49
2.3.2	The Power of Zero-Knowledge .....	51
2.3.3	The Role of Randomness .....	53
2.4	Probabilistically Checkable Proof Systems .....	53
2.4.1	Definition .....	53
2.4.2	The Power of Probabilistically Checkable Proofs .....	54
2.4.3	PCP and Approximation .....	57
2.4.4	More on PCP Itself .....	58
2.4.5	The Role of Randomness .....	60
2.5	Other Probabilistic Proof Systems .....	61
2.5.1	Restricting the Prover's Strategy .....	61
2.5.2	Non-Interactive Proofs .....	64
2.5.3	Proofs of Knowledge .....	64
2.5.4	Refereed Games .....	65
2.6	Concluding Remarks .....	65
2.6.1	Comparison Among the Various Notions .....	65
2.6.2	The Story .....	67
2.6.3	Open Problems .....	71
3.	Pseudorandom Generators .....	73
3.1	Introduction .....	73
3.2	The General Paradigm .....	75
3.3	The Archetypical Case .....	77
3.3.1	A Short Discussion .....	78
3.3.2	Some Basic Observations .....	79
3.3.3	Constructions .....	82
3.3.4	Pseudorandom Functions .....	85
3.4	Derandomization of Time-complexity Classes .....	87
3.5	Space Pseudorandom Generators .....	88
3.6	Special Purpose Generators .....	92
3.6.1	Pairwise-Independence Generators .....	93
3.6.2	Small-Bias Generators .....	95
3.6.3	Random Walks on Expanders .....	96
3.6.4	Samplers .....	98
3.6.5	Dispersers, Extractors and Weak Random Sources .....	101
3.7	Concluding Remarks .....	103
3.7.1	Discussion .....	104
3.7.2	Historical Perspective .....	104
3.7.3	Open Problems .....	106

<b>A. Background on Randomness and Computation</b> .....	107
A.1 Probability Theory – Three Inequalities .....	107
A.2 Computational Models and Complexity Classes .....	110
A.2.1 P, NP, and More .....	110
A.2.2 Probabilistic Polynomial-Time .....	111
A.2.3 Non-Uniform Polynomial-Time .....	113
A.2.4 Oracle Machines .....	115
A.2.5 Space Bounded Machines .....	116
A.2.6 Average-Case Complexity .....	117
A.3 Complexity Classes – Glossary .....	118
A.4 Some Basic Cryptographic Settings .....	119
A.4.1 Encryption Schemes .....	119
A.4.2 Digital Signatures and Message Authentication .....	121
A.4.3 The RSA and Rabin Functions .....	123
 <b>B. Randomized Computations</b> .....	125
B.1 Randomized Algorithms .....	125
B.1.1 Approx. Counting of DNF Satisfying Assignments .....	126
B.1.2 Finding a Perfect Matching .....	127
B.1.3 Testing Whether Polynomials Are Identical .....	130
B.1.4 Randomized Rounding Applied to MaxSAT .....	131
B.1.5 Primality Testing .....	132
B.1.6 Testing Graph Connectivity via a Random Walk .....	133
B.1.7 Finding Minimum Cuts in Graphs .....	134
B.2 Randomness in Complexity Theory .....	135
B.2.1 Reducing (Approximate) Counting to Deciding .....	135
B.2.2 Two-sided Error Versus One-sided Error .....	137
B.2.3 The Permanent: Worst-Case vs Average Case .....	138
B.3 Randomness in Distributed Computing .....	139
B.3.1 Testing String Equality .....	139
B.3.2 Routing in Networks .....	140
B.3.3 Byzantine Agreement .....	141
B.4 Bibliographic Notes .....	143
 <b>C. Two Proofs</b> .....	145
C.1 Parallel Repetition of Interactive Proofs .....	145
C.2 A Generic Hard-Core Predicate .....	149
C.2.1 A Motivating Discussion .....	151
C.2.2 Back to the Formal Argument .....	152
C.2.3 Improved Implementation of Algorithm <i>A'</i> .....	154
 <b>D. Related Surveys by the Author</b> .....	157
<b>Bibliography</b> .....	159
<b>Index</b> .....	179