

Contents

| | | |
|----------|--|----|
| 1 | Basics of spin geometry | 1 |
| 1.1 | Spin group and spin structure | 1 |
| 1.2 | Spinor bundle and Clifford multiplication | 4 |
| 1.3 | The Dirac operator | 9 |
| 1.4 | Spinors on hypersurfaces and coverings | 19 |
| 1.5 | Elliptic boundary conditions for the Dirac operator | 23 |
| 2 | Explicit computations of spectra | 29 |
| 2.1 | Spectrum of some non-negatively curved spaceforms | 29 |
| 2.2 | Spectrum of some other homogeneous spaces | 35 |
| 2.3 | Small eigenvalues of some symmetric spaces | 39 |
| 3 | Lower eigenvalue estimates on closed manifolds | 41 |
| 3.1 | Friedrich's inequality | 41 |
| 3.2 | Improving Friedrich's inequality in presence of a parallel form | 43 |
| 3.3 | Improving Friedrich's inequality in a conformal way | 55 |
| 3.4 | Improving Friedrich's inequality with the energy- momentum tensor | 58 |
| 3.5 | Improving Friedrich's inequality with other curvature components | 60 |
| 3.6 | Improving Friedrich's inequality on surfaces of positive genus | 61 |
| 3.7 | Improving Friedrich's inequality on bounding manifolds | 64 |
| 4 | Lower eigenvalue estimates on compact manifolds with boundary | 69 |
| 4.1 | Case of the gAPS boundary condition | 69 |
| 4.2 | Case of the CHI boundary condition | 70 |
| 4.3 | Case of the MIT bag boundary condition | 72 |
| 4.4 | Case of the mgAPS boundary condition | 74 |

| | |
|--|------------|
| 5 Upper eigenvalue bounds on closed manifolds | 77 |
| 5.1 Intrinsic upper bounds..... | 78 |
| 5.2 Extrinsic upper bounds | 85 |
| 6 Prescription of eigenvalues on closed manifolds | 93 |
| 6.1 Dirac isospectrality..... | 93 |
| 6.2 Harmonic spinors..... | 97 |
| 6.3 Prescribing the lower part of the spectrum..... | 100 |
| 7 The Dirac spectrum on non-compact manifolds | 103 |
| 7.1 Essential and point spectrum..... | 103 |
| 7.2 Explicit computations of spectra..... | 104 |
| 7.3 Lower bounds on the spectrum..... | 106 |
| 7.4 Absence of a spectral component | 109 |
| 8 Other topics related with the Dirac spectrum | 113 |
| 8.1 Other eigenvalue estimates | 113 |
| 8.2 Spectral gap | 116 |
| 8.3 Pinching Dirac eigenvalues | 117 |
| 8.4 Spectrum of other Dirac-type operators | 119 |
| 8.5 Conformal spectral invariants | 122 |
| 8.6 Convergence of eigenvalues | 125 |
| 8.7 Eta-invariants..... | 126 |
| 8.8 Positive mass theorems | 127 |
| A The twistor and Killing spinor equations..... | 131 |
| A.1 Definitions and examples | 131 |
| A.2 Elementary properties of twistor-spinors | 134 |
| A.3 Classification results for manifolds with twistor-spinors | 140 |
| A.4 Classification results for manifolds with Killing spinors..... | 141 |
| Bibliography | 145 |
| Index | 155 |