

Contents

1	Recurrent Problems	1
1.1	The Tower of Hanoi	1
1.2	Lines in the Plane	4
1.3	The Josephus Problem	8
	Exercises	17
2	Sums	21
2.1	Notation	21
2.2	Sums and Recurrences	25
2.3	Manipulation of Sums	30
2.4	Multiple Sums	34
2.5	General Methods	41
2.6	Finite and Infinite Calculus	47
2.7	Infinite Sums	56
	Exercises	62
3	Integer Functions	67
3.1	Floors and Ceilings	67
3.2	Floor/Ceiling Applications	70
3.3	Floor/Ceiling Recurrences	78
3.4	'mod': The Binary Operation	81
3.5	Floor/Ceiling Sums	86
	Exercises	95
4	Number Theory	102
4.1	Divisibility	102
4.2	Primes	105
4.3	Prime Examples	107
4.4	Factorial Factors	111
4.5	Relative Primality	115
4.6	'mod': The Congruence Relation	123
4.7	Independent Residues	126
4.8	Additional Applications	129
4.9	Phi and Mu	133
	Exercises	144
5	Binomial Coefficients	153
5.1	Basic Identities	153
5.2	Basic Practice	172
5.3	Tricks of the Trade	186

5.4	Generating Functions	196	
5.5	Hypergeometric Functions	204	
5.6	Hypergeometric Transformations	216	
5.7	Partial Hypergeometric Sums	223	
5.8	Mechanical Summation	229	
	Exercises	242	
6	Special Numbers		257
6.1	Stirling Numbers	257	
6.2	Eulerian Numbers	267	
6.3	Harmonic Numbers	272	
6.4	Harmonic Summation	279	
6.5	Bernoulli Numbers	283	
6.6	Fibonacci Numbers	290	
6.7	Continuants	301	
	Exercises	309	
7	Generating Functions		320
7.1	Domino Theory and Change	320	
7.2	Basic Maneuvers	331	
7.3	Solving Recurrences	337	
7.4	Special Generating Functions	350	
7.5	Convolutions	353	
7.6	Exponential Generating Functions	364	
7.7	Dirichlet Generating Functions	370	
	Exercises	371	
8	Discrete Probability		381
8.1	Definitions	381	
8.2	Mean and Variance	387	
8.3	Probability Generating Functions	394	
8.4	Flipping Coins	401	
8.5	Hashing	411	
	Exercises	427	
9	Asymptotics		439
9.1	A Hierarchy	440	
9.2	O Notation	443	
9.3	O Manipulation	450	
9.4	Two Asymptotic Tricks	463	
9.5	Euler's Summation Formula	469	
9.6	Final Summations	476	
	Exercises	489	
A	Answers to Exercises		497
B	Bibliography		604
C	Credits for Exercises		632
	Index		637
	List of Tables		657