Contents

Page

Notation VIII

Prologue 1

Chapter I. Lattices 9
1. Introduction 9
2. Bases and sublattices 9
3. Lattices under linear transformation 19
4. Forms and lattices 20
5. The polar lattice 23

Chapter II. Reduction 26
1. Introduction 26
2. The basic process 27
3. Definite quadratic forms 30
4. Indefinite quadratic forms 35
5. Binary cubic forms 51
6. Other forms 60

Chapter III. Theorems of BLICHFELDT and MINKOWSKI 64
1. Introduction 64
2. BLICHFELDT's and MINKOWSKI's theorems 68
3. Generalisations to non-negative functions 73
4. Characterisation of lattices 78
5. Lattice constants 80
6. A method of MORDELL 84
7. Representation of integers by quadratic forms 98

Chapter IV. Distance functions 103
1. Introduction 103
2. General distance-functions 105
3. Convex sets 108
4. Distance functions and lattices 119

Chapter V. MAHLER's compactness theorem 121
1. Introduction 121
2. Linear transformations 122
3. Convergence of lattices 126
4. Compactness for lattices 134
5. Critical lattices 141
6. Bounded star-bodies 145
7. Reducibility 152
8. Convex bodies 155
9. Spheres 163
10. Applications to diophantine approximation 165

Chapter VI. The theorem of MINKOWSKI-HLAWKA 175
1. Introduction 175
2. Sublattices of prime index 178

Page

3. The Minkowski-Hlawka theorem 181
4. SCHMIDT's theorems 184
5. A conjecture of ROGERS 187
6. Unbounded star-bodies 189

Chapter VII. The quotient space 194
1. Introduction 194
2. General properties 194
3. The sum theorem 198

Chapter VIII. Successive minima 201
1. Introduction 201
2. Spheres 205
3. General distance-functions 207
4. Convex sets 213
5. Polar convex bodies 219

Chapter IX. Packings 223
1. Introduction 223
2. Sets with $V(\mathcal{S}) = 2^n \Delta(\mathcal{S})$ 228
3. VORONOI's results 231
4. Preparatory lemmas 235
5. FEJES TÓTH's theorem 240
6. Cylinders 245
7. Packing of spheres 246
8. The product of n linear forms 250

Chapter X. Automorphs 256
1. Introduction 256
2. Special forms 266
3. A method of MORDELL 268
4. Existence of automorphs 279
5. Isolation theorems 286
6. Applications of isolation 295
7. An infinity of solutions 298
8. Local methods 301

Chapter XI. Inhomogeneous problems 303
1. Introduction 303
2. Convex sets 309
3. Transference theorems for convex sets 313
4. The product of n linear forms 322

Appendix 332

References 334

Index 343