

Contents

Algebraic Theory

Chapter I: Cohomology of Profinite Groups	3
§1. Profinite Spaces and Profinite Groups	3
§2. Definition of the Cohomology Groups	12
§3. The Exact Cohomology Sequence	25
§4. The Cup-Product	36
§5. Change of the Group G	45
§6. Basic Properties	60
§7. Cohomology of Cyclic Groups	74
§8. Cohomological Triviality	80
§9. Tate Cohomology of Profinite Groups	83
Chapter II: Some Homological Algebra	97
§1. Spectral Sequences	97
§2. Filtered Cochain Complexes	101
§3. Degeneration of Spectral Sequences	107
§4. The Hochschild-Serre Spectral Sequence	111
§5. The Tate Spectral Sequence	120
§6. Derived Functors	127
§7. Continuous Cochain Cohomology	136
Chapter III: Duality Properties of Profinite Groups	147
§1. Duality for Class Formations	147
§2. An Alternative Description of the Reciprocity Homomorphism	164
§3. Cohomological Dimension	171
§4. Dualizing Modules	181
§5. Projective pro- c -groups	189
§6. Profinite Groups of $scd G = 2$	202
§7. Poincaré Groups	210
§8. Filtrations	220
§9. Generators and Relations	224

Chapter IV: Free Products of Profinite Groups	245
§1. Free Products	245
§2. Subgroups of Free Products	252
§3. Generalized Free Products	256
Chapter V: Iwasawa Modules	267
§1. Modules up to Pseudo-Isomorphism	268
§2. Complete Group Rings	273
§3. Iwasawa Modules	289
§4. Homotopy of Modules	301
§5. Homotopy Invariants of Iwasawa Modules	312
§6. Differential Modules and Presentations	321

Arithmetic Theory

Chapter VI: Galois Cohomology	337
§1. Cohomology of the Additive Group	337
§2. Hilbert's Satz 90	343
§3. The Brauer Group	349
§4. The Milnor K -Groups	356
§5. Dimension of Fields	360
Chapter VII: Cohomology of Local Fields	371
§1. Cohomology of the Multiplicative Group	371
§2. The Local Duality Theorem	378
§3. The Local Euler-Poincaré Characteristic	391
§4. Galois Module Structure of the Multiplicative Group	401
§5. Explicit Determination of Local Galois Groups	409
Chapter VIII: Cohomology of Global Fields	425
§1. Cohomology of the Idèle Class Group	425
§2. The Connected Component of C_k	442
§3. Restricted Ramification	451
§4. The Global Duality Theorem	465
§5. Local Cohomology of Global Galois Modules	471
§6. Poitou-Tate Duality	479
§7. The Global Euler-Poincaré Characteristic	502
§8. Duality for Unramified and Tamely Ramified Extensions	512

Chapter IX: The Absolute Galois Group of a Global Field 521

- §1. The Hasse Principle 522
- §2. The Theorem of Grunwald-Wang 536
- §3. Construction of Cohomology Classes 543
- §4. Local Galois Groups in a Global Group 553
- §5. Solvable Groups as Galois Groups 557
- §6. Šafarevič's Theorem 574

Chapter X: Restricted Ramification 599

- §1. The Function Field Case 602
- §2. First Observations on the Number Field Case 618
- §3. Leopoldt's Conjecture 624
- §4. Cohomology of Large Number Fields 642
- §5. Riemann's Existence Theorem 647
- §6. The Relation between 2 and ∞ 656
- §7. Dimension of $H^i(G_S^T, \mathbb{Z}/p\mathbb{Z})$ 666
- §8. The Theorem of Kuz'min 678
- §9. Free Product Decomposition of $G_S(p)$ 686
- §10. Class Field Towers 697
- §11. The Profinite Group G_S 706

Chapter XI: Iwasawa Theory of Number Fields 721

- §1. The Maximal Abelian Unramified p -Extension of k_∞ 722
- §2. Iwasawa Theory for p -adic Local Fields 731
- §3. The Maximal Abelian p -Extension of k_∞ Unramified Outside S 735
- §4. Iwasawa Theory for Totally Real Fields and CM-Fields 751
- §5. Positively Ramified Extensions 763
- §6. The Main Conjecture 771

Chapter XII: Anabelian Geometry 785

- §1. Subgroups of G_k 785
- §2. The Neukirch-Uchida Theorem 791
- §3. Anabelian Conjectures 799

Literature 805

Index 820