

Contents

1	Continuum Formulation	1
1.1	General Aspects	1
1.2	Massless Spin Two Field	1
1.3	Wave Equation	2
1.4	Feynman Rules	11
1.5	One-Loop Divergences	16
1.6	Gravity in d Dimensions	21
1.7	Higher Derivative Terms	24
1.8	Supersymmetry	33
1.9	Supergravity	37
1.10	String Theory	40
1.11	Supersymmetric Strings	48
2	Feynman Path Integral Formulation	55
2.1	The Path Integral	55
2.2	Sum over Paths	56
2.3	Eulidean Rotation	58
2.4	Gravitational Functional Measure	59
2.5	Conformal Instability	64
3	Gravity in $2+\varepsilon$ Dimensions	67
3.1	Dimensional Expansion	67
3.2	Perturbatively Non-renormalizable Theories: The Sigma Model	68
3.3	Non-linear Sigma Model in the Large- N Limit	79
3.4	Self-coupled Fermion Model	83
3.5	The Gravitational Case	84
3.6	Phases of Gravity in $2+\varepsilon$ Dimensions	92
3.7	Running of $\alpha(\mu)$ in Gauge Theories	98

4	Hamiltonian and Wheeler-DeWitt Equation	103
4.1	Classical Initial Value Problem	103
4.2	First Order Formulation	104
4.3	Arnowitt-Deser-Misner (ADM) Formalism	107
4.4	Orthogonal Decomposition in the Linearized Theory	110
4.5	Intrinsic and Extrinsic Curvature, Hamiltonian	111
4.6	Matter Source Terms	112
4.7	Wheeler-DeWitt Equation	114
4.8	Semiclassical Expansion of the Wheeler-DeWitt Equation	115
4.9	Connection with the Feynman Path Integral	117
4.10	Minisuperspace	118
4.11	Solution of Simple Minisuperspace Models	122
4.12	Quantum Hamiltonian for Gauge Theories	127
4.13	Lattice Regularized Hamiltonian for Gauge Theories	129
4.14	Lattice Hamiltonian for Quantum Gravity	135
5	Semiclassical Gravity	141
5.1	Cosmological Wavefunctions	141
5.2	Semiclassical Expansion	146
5.3	Pair Creation in Constant Electric Fields	152
5.4	Black Hole Particle Emission	154
5.5	Method of In and Out Vacua	158
5.6	Complex Periodic Time	163
5.7	Black Hole Evaporation	166
5.8	Quantum Gravity Corrections	167
6	Lattice Regularized Quantum Gravity	169
6.1	The Lattice Theory	169
6.2	General Formulation	170
6.3	Volumes and Angles	171
6.4	Rotations, Parallel Transports and Voronoi Loops	173
6.5	Invariant Lattice Action	179
6.6	Lattice Diffeomorphism Invariance	184
6.7	Lattice Bianchi Identities	186
6.8	Gravitational Wilson Loop	188
6.9	Lattice Regularized Path Integral	190
6.10	An Elementary Example	195
6.11	Lattice Higher Derivative Terms	198
6.12	Scalar Matter Fields	203
6.13	Invariance Properties of the Scalar Action	209
6.14	Lattice Fermions, Tetrads and Spin Rotations	211
6.15	Gauge Fields	213
6.16	Lattice Gravitino	214
6.17	Alternate Discrete Formulations	216
6.18	Lattice Invariance versus Continuum Invariance	220

7	Analytical Lattice Expansion Methods	225
7.1	Motivation	225
7.2	Lattice Weak Field Expansion and Transverse-Traceless Modes	225
7.3	Lattice Diffeomorphism Invariance	234
7.4	Strong Coupling Expansion	242
7.5	Gravitational Wilson Loop	248
7.6	Discrete Gravity in the Large- d Limit	260
7.7	Mean Field Theory	269
8	Numerical Studies	273
8.1	Nonperturbative Gravity	273
8.2	Observables, Phase Structure and Critical Exponents	274
8.3	Invariant Local Gravitational Averages	276
8.4	Invariant Correlations at Fixed Geodesic Distance	278
8.5	Wilson Lines and Static Potential	281
8.6	Scaling in the Vicinity of the Critical Point	284
8.7	Physical and Unphysical Phases	285
8.8	Numerical Determination of the Scaling Exponents	290
8.9	Renormalization Group and Lattice Continuum Limit	295
8.10	Curvature Scales	300
8.11	Gravitational Condensate	302
9	Scale Dependent Gravitational Couplings	305
9.1	Renormalization Group and Scale Dependence of G	305
9.2	Effective Field Equations	305
9.3	Poisson's Equation and Vacuum Polarization Cloud	308
9.4	Static Isotropic Solution	311
9.5	Cosmological Solutions	314
9.6	Quantum Gravity and Mach's Principle	321
References		325
Index		335