

Contents

Preface *XI*

Introduction *XV*

1	Function Spaces, Linear Operators, and Green's Functions	1
1.1	Function Spaces	1
1.2	Orthonormal System of Functions	3
1.3	Linear Operators	5
1.4	Eigenvalues and Eigenfunctions	7
1.5	The Fredholm Alternative	9
1.6	Self-Adjoint Operators	12
1.7	Green's Functions for Differential Equations	14
1.8	Review of Complex Analysis	18
1.9	Review of Fourier Transform	25
2	Integral Equations and Green's Functions	31
2.1	Introduction to Integral Equations	31
2.2	Relationship of Integral Equations with Differential Equations and Green's Functions	37
2.3	Sturm–Liouville System	43
2.4	Green's Function for Time-Dependent Scattering Problem	47
2.5	Lippmann–Schwinger Equation	51
2.6	Scalar Field Interacting with Static Source	62
2.7	Problems for Chapter 2	67
3	Integral Equations of the Volterra Type	105
3.1	Iterative Solution to Volterra Integral Equation of the Second Kind	105
3.2	Solvable Cases of the Volterra Integral Equation	108
3.3	Problems for Chapter 3	112

4	Integral Equations of the Fredholm Type	117
4.1	Iterative Solution to the Fredholm Integral Equation of the Second Kind	117
4.2	Resolvent Kernel	120
4.3	Pincherle–Goursat Kernel	123
4.4	Fredholm Theory for a Bounded Kernel	127
4.5	Solvable Example	134
4.6	Fredholm Integral Equation with a Translation Kernel	136
4.7	System of Fredholm Integral Equations of the Second Kind	143
4.8	Problems for Chapter 4	143
5	Hilbert–Schmidt Theory of Symmetric Kernel	153
5.1	Real and Symmetric Matrix	153
5.2	Real and Symmetric Kernel	155
5.3	Bounds on the Eigenvalues	166
5.4	Rayleigh Quotient	169
5.5	Completeness of Sturm–Liouville Eigenfunctions	172
5.6	Generalization of Hilbert–Schmidt Theory	174
5.7	Generalization of the Sturm–Liouville System	181
5.8	Problems for Chapter 5	187
6	Singular Integral Equations of the Cauchy Type	193
6.1	Hilbert Problem	193
6.2	Cauchy Integral Equation of the First Kind	197
6.3	Cauchy Integral Equation of the Second Kind	201
6.4	Carleman Integral Equation	205
6.5	Dispersion Relations	211
6.6	Problems for Chapter 6	218
7	Wiener–Hopf Method and Wiener–Hopf Integral Equation	223
7.1	The Wiener–Hopf Method for Partial Differential Equations	223
7.2	Homogeneous Wiener–Hopf Integral Equation of the Second Kind	237
7.3	General Decomposition Problem	252
7.4	Inhomogeneous Wiener–Hopf Integral Equation of the Second Kind	261
7.5	Toeplitz Matrix and Wiener–Hopf Sum Equation	272
7.6	Wiener–Hopf Integral Equation of the First Kind and Dual Integral Equations	281
7.7	Problems for Chapter 7	285
8	Nonlinear Integral Equations	295
8.1	Nonlinear Integral Equation of the Volterra Type	295
8.2	Nonlinear Integral Equation of the Fredholm Type	299
8.3	Nonlinear Integral Equation of the Hammerstein Type	303
8.4	Problems for Chapter 8	305

9	Calculus of Variations: Fundamentals	309
9.1	Historical Background	309
9.2	Examples	313
9.3	Euler Equation	314
9.4	Generalization of the Basic Problems	319
9.5	More Examples	323
9.6	Differential Equations, Integral Equations, and Extremization of Integrals	326
9.7	The Second Variation	330
9.8	Weierstrass–Erdmann Corner Relation	345
9.9	Problems for Chapter 9	349
10	Calculus of Variations: Applications	353
10.1	Hamilton–Jacobi Equation and Quantum Mechanics	353
10.2	Feynman’s Action Principle in Quantum Theory	361
10.3	Schwinger’s Action Principle in Quantum Theory	368
10.4	Schwinger–Dyson Equation in Quantum Field Theory	371
10.5	Schwinger–Dyson Equation in Quantum Statistical Mechanics	385
10.6	Feynman’s Variational Principle	395
10.7	Poincare Transformation and Spin	407
10.8	Conservation Laws and Noether’s Theorem	411
10.9	Weyl’s Gauge Principle	418
10.10	Path Integral Quantization of Gauge Field I	437
10.11	Path Integral Quantization of Gauge Field II	454
10.12	BRST Invariance and Renormalization	468
10.13	Asymptotic Disaster in <i>QED</i>	475
10.14	Asymptotic Freedom in <i>QCD</i>	479
10.15	Renormalization Group Equations	487
10.16	Standard Model	499
10.17	Lattice Gauge Field Theory and Quark Confinement	518
10.18	WKB Approximation in Path Integral Formalism	523
10.19	Hartree–Fock Equation	526
10.20	Problems for Chapter 10	529
	References	567
	Index	573