Contents

Preface and Apology	vii
Chapter 1. Svlvester–Gallai Problem:	
The Beginnings of Combinatorial Geometry	1
1. James Joseph Sylvester and the Beginnings	1
2. Connecting Lines and Directions	3
3. Directions in Space vs. Points in the Plane	6
4. Proof of the Generalized Ungar Theorem	7
5. Colored Versions of the Sylvester–Gallai Theorem	10
Chapter 2. Arrangements of Surfaces:	
Evolution of the Basic Theory	13
1. Introduction	13
2. Preliminaries	16
3. Lower Envelopes	20
4. Single Cells	27
5. Zones	29
6. Levels	32
7. Many Cells and Related Problems	37
8. Generalized Voronoi Diagrams	40
9. Union of Geometric Objects	42
10. Decomposition of Arrangements	49
11. Representation of Arrangements	54
12. Computing Arrangements	56
13. Computing Substructures in Arrangements	58
14. Applications	63
15. Conclusions	70
Chapter 3. Davenport–Schinzel Sequences:	
The Inverse Ackermann Function in Geometry	73
1. Introduction	73
2. Davenport–Schinzel Sequences and Lower Envelopes	74
3. Simple Bounds and Variants	79
4. Sharp Upper Bounds on $\lambda_s(n)$	81
5. Lower Bounds on $\lambda_s(n)$	86
6. Davenport–Schinzel Sequences and Arrangements	89
Chapter 4. Incidences and Their Relatives:	
From Szemerédi and Trotter to Cutting Lenses	99
1. Introduction	99
2. Lower Bounds	102

3.	Upper Bounds for Incidences via the Partition Technique	104
4.	Incidences via Crossing Numbers—Székely's Method	106
5.	Improvements by Cutting into Pseudo-segments	109
6.	Incidences in Higher Dimensions	112
7.	Applications	114
Chan	ter 5 Crossing Numbers of Graphs	
Unap	Graph Drawing and its Applications	110
1	Crossings—the Brick Factory Problem	119
1. 2	Thrackles-Conway's Conjecture	119
2. 2	Different Crossing Numbers?	120
ა. 1	Charlest Line Dramin an	122
4. F	Angelan Develoption on d Classic	120
э. с	Angular Resolution and Slopes	120
ю. -	An Application in Computer Graphics	127
7.	An Unorthodox Proof of the Crossing Lemma	129
Chap	ter 6. Extremal Combinatorics:	
	Repeated Patterns and Pattern Recognition	133
1.	Models and Problems	133
2.	A Simple Sample Problem: Equivalence under Translation	135
3.	Equivalence under Congruence in the Plane	137
4.	Equivalence under Congruence in Higher Dimensions	139
5.	Equivalence under Similarity	141
6	Equivalence under Homothety or Affine Transformations	143
7.	Other Equivalence Relations for Triangles in the Plane	144
a		
Chap	ter 7. Lines in Space:	
-	From Ray Shooting to Geometric Transversals	147
1.	Introduction	147
2.	Geometric Preliminaries	149
3.	The Orientation of a Line Relative to n Given Lines	152
4.	Cycles and Depth Order	158
5.	Ray Shooting and Other Visibility Problems	163
6.	Transversal Theory	167
7.	Open Problems	170
Chap	ter 8 Geometric Coloring Problems:	
Chup	Sphere Packings and Frequency Allocation	173
1	Multiple Packings and Coverings	173
າ. ເ	Cover Decomposelle Families and Hunorgraph Colorings	175
2. 2	Enguancy Allocation and Conflict Free Coloring	170
ე. ⊿	Online Conflict Fine Coloning	1/0
4.	Online Connict-Free Coloring	181
Chap	ter 9. From Sam Loyd to László Fejes Tóth:	
	The 15 Puzzle and Motion Planning	183
1.	Sam Loyd and the Fifteen Puzzle	183
2.	Unlabeled Coins in Graphs and Grids	185
3.	László Fejes Tóth and Sliding Coins	187
4.	Pushing Squares Around	194
Bibliz	voranhy	107
	2 Prahuh	191
Index		227