Contents

Preface xiii
Contributors xvii
Color Insert (facing page 204)

I Wavelets and Wavelet Transforms 1

1 Wavelet Frames: Multiresolution Analysis and Extension Principles
John J. Benedetto, Oliver M. Treiber 3
1.1 Introduction . . . 3
1.2 Notation . . . 4
1.3 Some Properties of Frames . . . 7
1.4 The Frame Multiresolution Analysis Approach . . . 11
1.4.1 Frames generated by integer shifts of a function . . . 12
1.4.2 Frame decompositions from $\{\tau_k \phi : k \in \mathbb{Z}\}$ . . . 14
1.4.3 Frame multiresolution analysis (FMRA) . . . 16
1.4.4 Wavelet frames from frame multiresolution analyses . . . 17
1.5 A Generic Example of a Redundant Frame Generated by Integer Translates: Oversampling . . . 21
1.6 Perfect Reconstruction Multirate Systems from Frame Multiresolution Analyses . . . 24
1.7 Wavelet Frames for $L^2(\mathbb{R})$ from the Unitary Extension Principle of Ron and Shen . . . 26
1.7.1 The framework of the unitary extension principle . . . 27
1.7.2 Example: compactly supported tight spline frames . . . 28
1.7.3 Multirate systems from the unitary extension principle . . . 29
1.7.4 Proof of the extension principle . . . 30
References . . . 34

2 Convergence Rates of Multiscale and Wavelet Expansions
Mark A. Kon, Louise Arakelian Raphael **37**
2.1 Introduction and Definitions 37
2.2 Rates of Convergence and Wavelets 45
2.3 Proofs of Conditions on Scaling Functions 54
2.4 Proof of Theorems 2.1.1 and 2.1.2 62
2.5 Arbitrarily Slow Convergence 63
References 64

3 Denoising via Nonorthogonal Wavelet Transforms
Kathrin Berkner, Raymond O. Wells, Jr. **67**
3.1 Introduction 67
3.2 Maximal Decimated and Overcomplete Wavelet Transforms . . 68
3.3 Denoising via Nonlinear Processing in the Wavelet Domain . . 71
3.3.1 The Donoho–Johnstone method for denoising via thresholding of orthogonal wavelet coefficients—A review 72
3.3.2 Generalizations of the Donoho–Johnstone method to nonorthogonal DWT 74
3.4 Conclusions 78
References 79

4 Osiris Wavelets and the Dipole Gas
Guy Battle **81**
4.1 Introduction 81
4.2 Osiris Wavelets 92
4.3 A Positive Lower Bound on the Overlap Matrix 100
4.4 The Recursion Formula for the Dipole Gas 109
References 118

5 Wavelets in Closed Forms
Ahmed I. Zayed, Gilbert G. Walter **121**
5.1 Introduction 121
5.2 Preliminaries 123
5.3 Wavelet Construction 127
5.4 Orthonormal Wavelets in Closed Form 129
5.5 Interpolating Wavelets in Closed Form 138
References 142

6 Wavelet Galerkin Methods for Boundary Integral Equations and the Coupling with Finite Element Methods
Cristian Pérez, Reinhold Schneider **145**
6.1 Introduction 145
6.1.1 Coupling of finite elements with boundary integral methods (BEM–FEM coupling) 147

6.1.2 Biorthogonal wavelets and matrix compression 148
6.2 A Model Problem 149
6.3 The Coupling of Finite and Boundary Element Methods 151
6.3.1 Preliminaries 151
6.4 The Galerkin Scheme 153
6.4.1 A modified variational formulation 153
6.4.2 Strang's lemma and the effect of matrix compression 156
6.5 Biorthogonal Wavelets 157
6.5.1 Biorthogonal wavelet bases 157
6.6 Multiscale Methods and Matrix Compression 163
6.6.1 Basic estimates 163
6.6.2 Matrix compression 164
6.6.3 Matrix estimates 165
6.6.4 Consistency estimates 167
6.7 Matrix Compression for the Coupling of FEM–BEM 171
6.7.1 Biorthogonal wavelet bases for V_j and V_j' 171
6.7.2 Matrix compression for the bilinear form B 172
6.7.3 Consistency estimates 172
6.8 Convergence for the Compressed Coupling of FEM–BEM . . . 173
6.9 Complexity of the Compressed Coupling of FEM–BEM 175
References 177

7 Computing and Analyzing Turbulent Flows Using Wavelets
Kai Schneider, Marie Farge **181**
7.1 Introduction 181
7.2 Turbulence Computing 183
7.2.1 Governing equations 183
7.2.2 Numerical methods 187
7.2.3 Example 189
7.3 Statistical Analysis of Turbulent Flows 189
7.3.1 Experimental methodology 189
7.3.2 Averaging procedure 191
7.3.3 Prediction of the statistical theory 192
7.3.4 Classical statistical tools 193
7.3.5 Wavelet-based statistical tools 196
7.4 An Adaptive Wavelet Scheme 198
7.4.1 Time discretization 198
7.4.2 Spatial discretization 199
7.4.3 Two-dimensional case 202
7.4.4 Summary of the algorithm 202
7.4.5 Extension to the two-dimensional Navier–Stokes equations 202
7.5 Computation of Two-Dimensional Turbulent Flows 204
7.5.1 Temporally developing mixing layer 204

7.5.2 Decaying turbulence 206
7.5.3 Wavelet forced turbulence 207
7.6 Perspectives for Three-Dimensional Turbulent Flows 208
7.6.1 Vortex tube extraction in three-dimensional turbulence 210
7.7 Conclusions 211
References 212

8 The Uncertainty Principle for the Short-Time Fourier Transform and Wavelet Transform
Leon Cohen **217**
8.1 Introduction 217
8.2 Notation, Normalization, and the Standard Uncertainty Principle 218
8.3 Physical Quantities of the Spectrogram Related to Those of the Signal and Window 219
8.4 The Global Uncertainty Principle for the Spectrogram 220
8.5 Local Uncertainty Principle 222
8.6 Uncertainty Principle for Global–Local Quantities 224
8.6.1 Local duration—$\sigma_{\omega|t}$ relation 224
8.6.2 Local bandwidth—$\sigma_{t|\omega}$ relation 225
8.7 The Uncertainty Principle for "Scale-Time" 226
8.8 The Wavelet Transform and Scalogram 227
8.9 Marginals and Moments of the Scalogram 228
8.9.1 Moments for the scalogram. 229
8.9.2 Time Moments 229
8.9.3 Frequency Moments 230
8.10 Conclusion 231
References 231

II Time-Frequency Signal Analysis **233**

9 Quadratic Time-Frequency Analysis of Linear Time-Varying Systems
Franz Hlawatsch, Gerald Matz **235**
9.1 Introduction 235
9.1.1 Background and motivation 236
9.1.2 Outline 237
9.1.3 Elements of LTV system theory 238
9.1.4 Spreading function and Weyl symbol 240
9.2 The Transfer Wigner Distribution 240
9.2.1 Energetic interpretation 241
9.2.2 Properties 242
9.2.3 Examples 243

9.3 The Input Wigner Distribution 246
9.3.1 Energetic interpretation 246
9.3.2 Expressions 248
9.3.3 Properties 249
9.3.4 Examples 252
9.4 The Output Wigner Distribution 252
9.4.1 Energetic interpretation 253
9.4.2 Expressions 255
9.4.3 Properties 255
9.4.4 Examples 257
9.5 Time-Frequency Weighting and Displacement 258
9.5.1 Characterization of time-frequency weighting by the IWD and OWD 258
9.5.2 Centroids and spreads 259
9.6 Normal Systems 262
9.6.1 Time-frequency description 262
9.6.2 Classes and examples of normal systems 263
9.6.3 Systems with minimum time-frequency displacement 267
9.6.4 Simulation results 268
9.7 Approximations for Underspread Systems 269
9.7.1 Equivalence of IWD, OWD, and the squared Weyl symbol 270
9.7.2 Positivity 272
9.7.3 Composition property 273
9.8 Random LTV Systems 274
9.9 Time-Frequency Design of LTV Systems 276
9.10 Conclusion 278
Appendix A: Minimization of the Time-Frequency Displacement Spread 280
Appendix B: Proof of Underspread Approximations 280
Appendix C: Solution of the TF Design Problem 282
References 283

10 Inequalities in Mellin–Fourier Signal Analysis
Patrick Flandrin **289**
10.1 Introduction 289
10.2 Inequalities for Scale Transforms 292
10.2.1 Variance inequalities 292
10.2.2 Modified variance inequalities 293
10.2.3 Entropy inequalities 297
10.2.4 Narrowband limit 299
10.3 Inequalities on the Scale-Frequency Plane 300
10.3.1 Joint distributions of scale and frequency 302
10.3.2 Variance-type inequalities 304

10.3.3 An uncertainty relation for the wavelet transform . . . 306
10.3.4 An entropy inequality for the unitary Bertrand distribution . . . 312
10.4 Conclusion . . . 316
References . . . 317

11 Introduction to Time-Frequency Signal Analysis
Boualem Boashash, Braham Barkat **321**
11.1 Introduction . . . 321
11.2 Fundamental Signal Representations . . . 323
11.2.1 Signal models . . . 323
11.2.2 Need for a joint time-frequency analysis . . . 325
11.2.3 Signal characteristics . . . 328
11.3 Review of Contributions to Time-Frequency Signal Analysis . 331
11.3.1 The early theoretical contributions . . . 331
11.3.2 The second phase of advances in TFSA . . . 337
11.4 Quadratic Time-Frequency Distributions . . . 340
11.4.1 A derivation procedure . . . 340
11.4.2 Time, frequency, lag, and Doppler representations of the quadratic class of TFDs . . . 341
11.4.3 Multicomponent signal analysis . . . 343
11.4.4 Discrete-time implementation of quadratic TFDs . . . 354
11.5 Time-Frequency Analysis of Nonlinear FM Signals . . . 357
11.5.1 Polynomial FM signals . . . 357
11.5.2 Optimality of the WVD for linear FM signals . . . 357
11.5.3 Design of polynomial Wigner–Ville distributions . . . 359
11.5.4 Multicomponent signals and polynomial TFDs . . . 364
11.5.5 IF estimation using the PWVD . . . 368
References . . . 374

12 Reduced Interference Time-Frequency Distributions: Scaled Decompositions and Interpretations
William J. Williams **381**
12.1 Introduction . . . 381
12.1.1 Philosophies of Cohen's class . . . 382
12.2 The Reduced Interference Distribution . . . 383
12.2.1 Ambiguity function relationships . . . 383
12.2.2 The exponential distribution . . . 384
12.2.3 Design procedures for effective RID kernels . . . 384
12.3 Discrete Formulations and Fast Algorithms . . . 388
12.3.1 Discrete realizations . . . 389
12.4 Applications and Interpretations of RID . . . 391
12.4.1 Bioacoustic applications of RID . . . 396
12.5 Wavelet Approaches . . . 398
12.6 Complete Time-Frequency Bases . . . 400

12.7 Decomposition of Time-Frequency Distributions 400
12.7.1 Representation of TFDs using linear operator notation 401
12.7.2 Spectrogram decomposition of discrete TFDs 401
12.7.3 Decomposition using scaled windows 403
12.8 Kernel Decomposition Results 406
12.8.1 Spectrogram decomposition results 410
12.9 Discussion 410
12.10 Conclusions 413
References 414

Index **419**