Contents

1 **Introduction and Summary** 1
1.1 Introduction 1
1.2 Preliminary Examples 1
1.3 Summary of the Following Chapters 6

2 **Continuous-Time Markov Decision Processes** 9
2.1 Introduction 9
2.2 The Control Model 10
2.3 Continuous-Time Markov Decision Processes 13
2.4 Basic Optimality Criteria 16

3 **Average Optimality for Finite Models** 19
3.1 Introduction 19
3.2 n-bias Optimality Criteria 20
3.3 Difference Formulas of n-biases 23
3.4 Characterization of n-bias Policies 29
3.5 Computation of n-bias Optimal Policies 36
3.5.1 The Policy Iteration Algorithm for Average Optimality . . . 36
3.5.2 The 0-bias Policy Iteration Algorithm 39
3.5.3 n-bias Policy Iteration Algorithms 43
3.6 The Linear Programming Approach 46
3.6.1 Linear Programming for Ergodic Models 46
3.6.2 Linear Programming for Multichain Models 49
3.7 Notes 52

4 **Discount Optimality for Nonnegative Costs** 55
4.1 Introduction 55
4.2 The Nonnegative Model 55
4.3 Preliminaries 56
4.4 The Discounted Cost Optimality Equation 60
4.5 Existence of Optimal Policies 63
4.6 Approximation Results 63

Bibliografische Informationen
http://d-nb.info/994394683

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

4.7 The Policy Iteration Approach . . . 66
4.8 Examples . . . 68
4.9 Notes . . . 69

5 Average Optimality for Nonnegative Costs . . . 71
5.1 Introduction . . . 71
5.2 The Average-Cost Criterion . . . 72
5.3 The Minimum Nonnegative Solution Approach . . . 73
5.4 The Average-Cost Optimality Inequality . . . 76
5.5 The Average-Cost Optimality Equation . . . 80
5.6 Examples . . . 81
5.7 Notes . . . 84

6 Discount Optimality for Unbounded Rewards . . . 87
6.1 Introduction . . . 87
6.2 The Discounted-Reward Optimality Equation . . . 89
6.3 Discount Optimal Stationary Policies . . . 95
6.4 A Value Iteration Algorithm . . . 98
6.5 Examples . . . 98
6.6 Notes . . . 102

7 Average Optimality for Unbounded Rewards . . . 105
7.1 Introduction . . . 105
7.2 Exponential Ergodicity Conditions . . . 106
7.3 The Existence of AR Optimal Policies . . . 109
7.4 The Policy Iteration Algorithm . . . 113
7.5 Examples . . . 119
7.6 Notes . . . 124

8 Average Optimality for Pathwise Rewards . . . 127
8.1 Introduction . . . 127
8.2 The Optimal Control Problem . . . 129
8.3 Optimality Conditions and Preliminaries . . . 129
8.4 The Existence of PAR Optimal Policies . . . 131
8.5 Policy and Value Iteration Algorithms . . . 138
8.6 An Example . . . 139
8.7 Notes . . . 142

9 Advanced Optimality Criteria . . . 143
9.1 Bias and Weakly Overtaking Optimality . . . 143
9.2 Sensitive Discount Optimality . . . 147
9.3 Blackwell Optimality . . . 159
9.4 Notes . . . 160

10 Variance Minimization . . . 163
10.1 Introduction . . . 163

10.2 Preliminaries . . . 164
10.3 Computation of the Average Variance . . . 164
10.4 Variance Minimization . . . 170
10.5 Examples . . . 171
10.6 Notes . . . 173

11 Constrained Optimality for Discount Criteria . . . 175
11.1 The Model with a Constraint . . . 175
11.2 Preliminaries . . . 177
11.3 Proof of Theorem 11.4 . . . 182
11.4 An Example . . . 184
11.5 Notes . . . 186

12 Constrained Optimality for Average Criteria . . . 187
12.1 Average Optimality with a Constraint . . . 187
12.2 Preliminaries . . . 188
12.3 Proof of Theorem 12.4 . . . 192
12.4 An Example . . . 192
12.5 Notes . . . 194

A . . . 195
A.1 Limit Theorems . . . 195
A.2 Results from Measure Theory . . . 197

B . . . 203
B.1 Continuous-Time Markov Chains . . . 203
B.2 Stationary Distributions and Ergodicity . . . 206

C . . . 209
C.1 The Construction of Transition Functions . . . 209
C.2 Ergodicity Based on the Q-Matrix . . . 214
C.3 Dynkin's Formula . . . 218

References . . . 221

Index . . . 229