CONTENTS

Foreword		oage xiii		
Par	t I	Finite Dimensional Control Problems	1	
1	Cal	culus of Variations and Control Theory	3	
	1.1	Calculus of Variations: Surface of Revolution of		
		Minimum Area	3	
	1.2	Interpretation of the Results	8	
	1.3	Mechanics and Calculus of Variations	9	
	1.4	Optimal Control: Fuel Optimal Landing of a Space Vehicle	11	
	1.5	Optimal Control Problems Described by Ordinary		
		Differential Equations	12	
	1.6	Calculus of Variations and Optimal Control.		
		Spike Perturbations	13	
	1.7	Optimal Control: Minimum Drag Nose Shape in		
		Hypersonic Flow	17	
	1.8	Control of Functional Differential Equations:		
		Optimal Forest Growth	18	
	1.9	Control of Partial Differential Equations	20	
	1.10) Finite Dimensional and Infinite Dimensional Control Problem	ns 25	
2	Optimal Control Problems Without Target Conditions 24			
	2.0	Elements of Measure and Integration Theory	26	
	2.1	Control Systems Described by Ordinary Differential Equation	ns 42	
	2.2	Existence Theory for Optimal Control Problems	51	
	2.3	Trajectories and Spike Perturbations	60	
	2.4	Cost Functionals and Spike Perturbations	66	
	2.5	Optimal Control Problems without Target Condition:		
		The Hamiltonian Formalism	67	

	2.6	Invariance of the Hamiltonian	71
	2.7	The Linear-Quadratic Problem: Existence and Uniqueness	
		of Optimal Controls	76
	2.8	The Unconstrained Linear-Quadratic Problem: Feedback,	
		the Riccati Equation	78
	2.9	The Constrained Linear-Quadratic Problem	82
3	Abst	act Minimization Problems: The Minimum Principle	
	for th	e Time Optimal Problem	84
	3.1	Abstract Minimization Problems	84
	3.2	Ekeland's Variational Principle	87
	3.3	The Abstract Time Optimal Problem	92
	3.4	The Control Spaces	100
	3.5	Continuity of the Solution Map	101
	3.6	Continuity of the Solution Operator of the	
		Variational Equation	102
	3.7	The Minimum Principle for the Time Optimal Problem	104
	3.8	Time Optimal Capture of a Wandering Particle	107
	3.9	Time Optimal Stopping of an Oscillator	111
	3.10	Higher Dimensional Problems	118
4	The l	Minimum Principle for General Optimal Control Problems	122
	4.1	The Abstract Minimization Problem	122
	4.2	The Minimum Principle for Problems with Fixed	
		Terminal Time	125
	4.3	Optimal Capture of a Wandering Particle in Fixed Time, I	129
	4.4	Singular Intervals and Singular Arcs	137
	4.5	Optimal Capture of a Wandering Particle in Fixed Time, II	138
	4.6	The Minimum Principle for Problems with Variable	
		Terminal Time	143
	4.7	Fuel Optimal Soft Landing of a Space Vehicle	146
	4.8	Fuel Optimal Soft Landing of a Space Vehicle	149
	4.9	The Linear-Quadratic Problem and the Minimum Drag	
		Nose Shape Problem	151
	4.10	Nonlinear Programming Problems: The Kuhn-Tucker	
		Theorem	159
Par	tII l	Infinite Dimensional Control Problems	167
5	Diffe	rential Equations in Banach Spaces and Semigroup Theory	169
	5.0	Banach Spaces and Their Duals. Linear Operators.	
		Integration of Vector Valued Functions	169
	5.1	Partial Differential Equations as Ordinary Differential	
		Equations in Banach Spaces	189

	5.2	Abstract Cauchy Problems in $t \ge 0$	192	
	5.3	Abstract Cauchy Problems in $-\infty < t < \infty$	203	
	5.4	Evolution Equations	207	
	5.5	Semilinear Equations in Banach Spaces. Perturbation Theory	210	
	5.6	Wave Equations	226	
	5.7	Semilinear Wave Equations: Local Existence	234	
	5.8	Semilinear Equations in Banach Spaces: Global Existence	239	
	5.9	Semilinear Wave Equations: Global Existence	246	
6	Abstract Minimization Problems in Hilbert Spaces 25			
	6.1	Control Systems: Continuity of the Solution Map	251	
	6.2	Patch Perturbations and Directional Derivatives	253	
	6.3	Continuity of the Solution Operator of the		
		Variational Equation	262	
	6.4	Abstract Minimization Problems Again	263	
	6.5	The Minimum Principle for the Time Optimal Problem	273	
	6.6	The Minimum Principle for General Control Problems	279	
	6.7	Optimal Problems for Some Linear and Semilinear Equations	283	
	6.8	Semilinear Wave Equations Again	288	
	6.9	The Time Optimal Problem for a Semilinear		
		Wave Equation, I	291	
	6.10	Some Remarks on Adjoint Equations	293	
	6.11	Some Remarks on Controllability	298	
	6.12	The Time Optimal Problem for a Semilinear		
		Wave Equation, II	303	
7	Absti	ract Minimization Problems in Banach Spaces	310	
	7.1	Some Geometry of Banach Spaces	310	
	7.2	Abstract Minimization Problems for the Last Time	316	
	7.3	The Minimum Principle in Banach Spaces	325	
	7.4	Fractional Powers of Infinitesimal Generators.		
		Analytic Semigroups. Duality	329	
	7.5	Elliptic Operators in L^2 Spaces	340	
	7.6	Elliptic Operators in L^p and C Spaces. Duality	343	
	7.7	Semilinear Abstract Parabolic Equations	347	
	7.8	Semilinear Abstract Parabolic Equations: Global Existence	359	
	7.9	Linear Abstract Parabolic Equations. Duality	364	
	7.10	Patch Perturbations and Directional Derivatives	377	
8		polation and Domains of Fractional Powers	385	
	8.1	Trace Spaces and Semigroups	385	
	8.2	Interpolation and Fractional Powers	393	
	8.3	Interpolation and Sobolev Spaces	399	

	8.4	Parabolic Equations	402
	8.5	Fractional Powers and the Complex Interpolation Method	412
	8.6	The Navier-Stokes Equations	418
9	Linear Control Systems		426
	9.1	Linear Systems: The Minimum Principle	426
	9.2	The Minimum Principle with Full Control	437
	9.3	Bang-Bang Theorems and Approximate Controllability	444
	9.4	Exact and Approximate Controllability	452
	9.5	Controllability with Finite Dimensional Controls	458
	9.6	Controllability and the Minimum Principle	467
10	Optim	al Control Problems with State Constraints	474
	10.1	Optimal Control Problems with State Constraints	474
	10.2	Integration with Respect to Vector-Valued Measures	475
	10.3	The Minimum Principle with State Constraints	490
	10.4	Saturation of the State Constraint	498
	10.5	Surface of Revolution of Minimum Area as a Control	
		Problem	501
	10.6	Other Applications	506
11	Optim	al Control Problems with State Constraints	509
	11.1	Abstract Parabolic Equations: The Measure-Driven	
		Adjoint Variational Equation	509
	11.2	Abstract Parabolic Equations: The Minimum Principle	
		with State Constraints	517
	11.3	Applications to Parabolic Distributed Parameter Systems	523
	11.4	Parabolic Distributed Parameter Systems, I	528
	11.5	Parabolic Distributed Parameter Systems, II	537
	11.6	Linear Systems: The Minimum Principle with State	
		Constraints	548
	11.7	Control Problems for the Navier-Stokes Equations	558
	11.8	Control Problems for the Navier–Stokes Equations:	
		The Point Target Case	562
	11.9	Convergence of Suboptimal Controls, I	564
	11.10	Convergence of Suboptimal Controls, II	570
	11.11	Parabolic Equations	575
	11.12	The Navier–Stokes Equations	583
Par	rt III F	Relaxed Controls	601
12		s of Relaxed Controls. Topology and Measure Theory	603
	12.0	Weak Topologies in Linear Spaces	603
	12.1	Existence Theory of Optimal Control Problems:	
		Measure-Valued Controls	614

	12.2	Spaces of Vector Valued Functions and Their Duals, I	618
	12.3	Finitely Additive Measures: Integration	628
	12.4	Measures and Linear Functionals in Function Spaces	635
	12.5	Spaces of Relaxed Controls	642
	12.6	Approximation in Spaces of Measures and Spaces	
		of Relaxed Controls	648
	12.7	Topology and Measure Theory	654
	12.8	The Filippov Implicit Function Theorem	659
	12.9	Spaces of Vector Valued Functions and Their Duals, II	666
13	Relaxe	ed Controls in Finite Dimensional Systems	674
	13.1	Installation of Relaxed Controls in Finite	
		Dimensional Systems	674
	13.2	Approximation of Relaxed Trajectories	
		by Ordinary Trajectories	677
	13.3	The Filippov Implicit Function Theorem in	
		the Compact Case	679
	13.4	Differential Inclusions	683
	13.5	Existence Theorems for Relaxed Optimal Control Problems	685
	13.6	Existence Theorems for Ordinary Optimal Control Problems	690
	13.7	The Minimum Principle for Relaxed Optimal	
		Control Problems	692
	13.8	Noncompact Control Sets	699
14	Relaxe	ed Controls in Infinite Dimensional Systems	709
	14.1	Control Systems: Limits of Trajectories	709
	14.2	Semilinear Systems Linear in the Control. Approximation	
		by Extremal Trajectories	711
	14.3	Installation of Relaxed Controls in Infinite Dimensional	
		Systems	720
	14.4	Differential Inclusions	725
	14.5	Existence Theorems for Optimal Control Problems, I	730
	14.6	Existence Theorems for Optimal Control Problems, II	738
	14.7	Abstract Parabolic Equations, I	742
	14.8	Abstract Parabolic Equations, II	748
	14.9	Existence Under Compactness of the Nonlinear Term	754
	14.10	Existence Without Compactness	759
Ref	erences		773
	References Index		795