

Contents

List of symbols	XIII
1 Introduction	1
A Fundamentals of elasticity	5
– Chapter 2 to 7 –	
A.1 Definitions – Formulas – Concepts	5
2 Tensor algebra and analysis	5
2.1 Terminology – definitions	5
2.2 Index rules and summation convention	6
2.3 Tensor of first order (vector)	7
2.4 Tensors of second and higher order	10
2.5 Curvilinear coordinates	13
3 State of stress	18
3.1 Stress vector	18
3.2 Stress tensor	20
3.3 Coordinate transformation – principal axes	21
3.4 Stress deviator	24
3.5 Equilibrium conditions	25
4 State of strain	26
4.1 Kinematics of a deformable body	26
4.2 Strain tensor	29
4.3 Strain–displacement relations	30
4.4 Transformation of principal axes	31
4.5 Compatibility conditions	31
5 Constitutive laws of linearly elastic bodies	31
5.1 Basic concepts	31
5.2 Generalized HOOKE–DUHAMEL's law	32
5.3 Material law for plane states	35
5.4 Material law for a unidirectional layer (UD-layer) of a fibre reinforced composite	37

6	Energy principles	39
6.1	Basic terminology and assumptions	39
6.2	Energy expressions	40
6.3	Principle of virtual displacements (Pvd)	44
6.4	Principle of virtual forces (Pvf)	44
6.5	Reciprocity theorems and <i>Unit-Load-Method</i>	46
6.6	Treatment of a variational problem	46
6.7	Approximation methods for continua	47
7	Problem formulations in the theory of linear elasticity	48
7.1	Basic equations and boundary-value problems	48
7.2	Solution of basic equations	49
7.3	Special equations for three-dimensional problems	49
7.4	Special equations for plane problems	50
7.5	Comparison of <i>state of plane stress</i> and <i>state of plane strain</i>	51
A.2	Exercises	53
A-2-1:	Tensor rules in oblique base	53
A-2-2:	Analytical vector expressions for a parallelogram disk	60
A-2-3:	Analytical vector expressions for an elliptical hole in elliptical-hyperbolical coordinates	63
A-3-1:	MOHR'S circle for a state of plane stress	66
A-3-2:	Principal stresses and axes of a three-dimensional state of stress	67
A-3-3:	Equilibrium conditions in elliptical-hyperbolical coordinates (continued from A-2-3)	70
A-4-1:	Displacements and compatibility of a rectangular disk	71
A-4-2:	Principal strains from strain gauge measurements	73
A-4-3:	Strain tensor, principal strains and volume dilatation of a three-dimensional state of displacements	74
A-4-4:	Strain-displacement relation and material law in elliptical-hyperbolical coordinates (continued from A-2-3)	76
A-5-1:	Steel ingot in a rigid concrete base	78
A-6-1:	Differential equation and boundary conditions for a BERNOULLI beam from a variational principle	80
A-6-2:	Basic equations of linear thermoelasticity by HELLINGER/REISSNER'S variational functional	82
A-6-3:	Application of the principle of virtual displacements for establishing the relations of a triangular, finite element	83
A-7-1:	Hollow sphere under constant inner pressure	86
A-7-2:	Single load acting on an elastic half-space - Application of LOVE'S displacement function	89

B	Plane load-bearing structures	93
	– Chapter 8 to 10 –	
B.1	Definitions – Formulas – Concepts	93
8	Disks	93
8.1	Definitions – Assumptions – Basic Equations	93
8.2	Analytical solutions to the homogeneous bipotential equation	95
9	Plates	99
9.1	Definitions – Assumptions – Basic Equations	99
9.2	Analytical solutions for shear-rigid plates	107
10	Coupled disk-plate problems	113
10.1	Isotropic plane structures with large displacements	113
10.2	Load-bearing structures made of composite materials	118
B.2	Exercises	123
B-8-1:	Simply supported rectangular disk under constant load	123
B-8-2:	Circular annular disk subjected to a stationary temperature field	128
B-8-3:	Rotating solid and annular disk	131
B-8-4:	Clamped quarter-circle disk under a single load	133
B-8-5:	Semi-infinite disk subjected to a concentrated moment	137
B-8-6:	Circular annular CFRP-disk under several loads	139
B-8-7:	Infinite disk with an elliptical hole under tension	145
B-8-8:	Infinite disk with a crack under tension	151
B-9-1:	Shear-rigid, rectangular plate subjected to a triangular load	153
B-9-2:	Shear-stiff, semi-infinite plate strip under a boundary moment	155
B-9-3:	Rectangular plate with two elastically supported boundaries subjected to a temperature gradient field	157
B-9-4:	Overall clamped rectangular plate under a constantly distributed load	167
B-9-5:	Rectangular plate with mixed boundary conditions under distributed load	170
B-9-6:	Clamped circular plate with a constant circular line load	172
B-9-7:	Clamped circular ring plate with a line load at the outer boundary	177
B-9-8:	Circular plate under a distributed load rested on an elastic foundation	179

B-9-9: Centre-supported circular plate with variable thickness under constant pressure load.	183
B-10-1: Buckling of a rectangular plate with one stiffener	188
B-10-2: Clamped circular plate under constant pressure considered as a coupled disk-plate problem	195
C Curved load-bearing structures	199
– Chapter 11 to 14 –	
C.1 Definitions – Formulas – Concepts	199
11 General fundamentals of shells	199
11.1 Surface theory – description of shells	199
11.2 Basic theory of shells	209
11.3 Shear-rigid shells with small curvature	213
12 Membrane theory of shells	214
12.1 General basic equations	214
12.2 Equilibrium conditions of shells of revolution	215
12.3 Equilibrium conditions of translation shells	218
12.4 Deformations of shells of revolution	220
12.5 Constitutive equations – material law	221
12.6 Specific deformation energy	221
13 Bending theory of shells of revolution	222
13.1 Basic equations for arbitrary loads	222
13.2 Shells of revolution with arbitrary meridional shape – Transfer Matrix Method	228
13.3 Bending theory of a circular cylindrical shell	233
14 Theory of shallow shells	241
14.1 Characteristics of shallow shells	241
14.2 Basic equations and boundary conditions	242
14.3 Shallow shell over a rectangular base with constant principal curvatures	245
C.2 Exercises	247
C-11-1: Fundamental quantities and equilibrium conditions of the membrane theory of a circular conical shell	247
C-12-1: Shell of revolution with elliptical meridional shape subjected to constant internal pressure	251
C-12-2: Spherical boiler under internal pressure and centrifugal force	253

C-12-3: Spherical shell under wind pressure	255
C-12-4: Hanging circular conical shell filled with liquid	258
C-12-5: Circular toroidal ring shell subjected to a uniformly distributed boundary load	260
C-12-6: Circular cylindrical cantilever shell subjected to a transverse load at the end	264
C-12-7: Skew hyperbolic paraboloid (<i>hypar shell</i>) subjected to deadweight	267
C-13-1: Water tank with variable wall thickness under liquid pressure	272
C-13-2: Cylindrical pressure tube with a shrunk ring	276
C-13-3: Pressure boiler	281
C-13-4: Circular cylindrical shell horizontally clamped at both ends subjected to deadweight	283
C-13-5: Buckling of a cylindrical shell under external pressure	288
C-13-6: Free vibrations of a circular cylindrical shell	290
C-14-1: Spherical cap under a concentrated force at the vertex	293
C-14-2: Eigenfrequencies of a hypar shell	296

D Structural optimization 301

– Chapter 15 to 18 –

D.1 Definitions – Formulas – Concepts 301

15 Fundamentals of structural optimization	301
15.1 Motivation – aim – development	301
15.2 Single problems in a design procedure	302
15.3 Design variables – constraints – objective function	303
15.4 Problem formulation – task of structural optimization	306
15.5 Definitions in mathematical optimization	307
15.6 Treatment of a Structural Optimization Problem (SOP)	309
16 Algorithms of Mathematical Programming (MP)	310
16.1 Problems without constraints	310
16.2 Problems with constraints	314
17 Sensitivity analysis of structures	321
17.1 Purpose of sensitivity analysis	321
17.2 Overall Finite Difference (OFD) sensitivity analysis	322
17.3 Analytical and semi-analytical sensitivity analyses	322

18 Optimization strategies	325
18.1 Vector, multiobjective or multicriteria optimization – PARETO-optimality	325
18.2 Shape optimization	329
18.3 Augmented optimization loop by additional strategies	334
D.2 Exercises	337
D-15/16-1: Exact and approximate solution of an unconstrained optimization problem	337
D-15/16-2: Optimum design of a plane truss structure – sizing problem	342
D-15/16-3: Optimum design of a part of a long circular cylindrical boiler with a ring stiffener – sizing problem	347
D-18-1: Mathematical treatment of a Vector Optimization Problem	352
D-18-2: Simply supported column – shape optimization problem by means of calculus of variations	355
D-18-3: Optimal design of a conveyor belt drum – use of shape functions	360
D-18-4: Optimal shape design of a satellite tank – treatment as a multicriteria optimization problem	364
D-18-5: Optimal layout of a point-supported sandwich panel made of CFRP-material – geometry optimization	370
References	375
A Fundamentals of elasticity	375
B Plane load-bearing structures	376
C Curved load-bearing structures	377
D Structural optimization	378
Subject index	383