CONTENTS

PREFACE

CHAPTER 1. INTRODUCTION 1
1.1 Elastic Behavior 2
1.2 Plastic Behavior 2
1.3 Viscoelastic Behavior 2
1.4 Creep 3
1.5 Recovery 4
1.6 Relaxation 5
1.7 Linearity 5

CHAPTER 2. HISTORICAL SURVEY OF CREEP 8
2.1 Creep of Metals 8
2.2 Creep under Uniaxial Stress 8
2.3 Creep under Combined Stresses 11
2.4 Creep under Variable Stress 13
2.5 Creep of Plastics 13
2.6 Mathematical Representation of Creep of Materials 15
2.7 Differential Form 15
2.8 Integral Form 17
2.9 Development of Nonlinear Constitutive Relations 17

CHAPTER 3. STATE OF STRESS AND STRAIN 22
3.1 State of Stress 22
3.2 Stress Tensor 23
3.3 Unit Tensor 23
3.4 Principal Stresses 24
3.5 Mean Normal Stress Tensor and Deviatoric Stress Tensor 26
3.6 Invariants of Stress 30
3.7 Traces of Tensors and Products of Tensors 30
3.8 Invariants in Terms of Traces 32
3.9 Hamilton–Cayley Equation 33
3.10 State of Strain 35
3.11 Strain-Displacement Relation 37
3.12 Strain Tensor 38

CHAPTER 4. MECHANICS OF STRESS AND DEFORMATION ANALYSES 40
4.1 Introduction 40
4.2 Law of Motion 41

4.3 Equations of Equilibrium 41
4.4 Equilibrium of Moments 43
4.5 Kinematics 43
4.6 Compatibility Equations 43
4.7 Constitutive Equations 45
4.8 Linear Elastic Solid 46
4.9 Boundary Conditions 48
4.10 The Stress Analysis Problem in a Linear Isotropic Elastic Solid 48

CHAPTER 5. LINEAR VISCOELASTIC CONSTITUTIVE EQUATIONS 50
5.1 Introduction 50
5.2 Viscoelastic Models 51
5.3 The Basic Elements: Spring and Dashpot 52
5.4 Maxwell Model 53
5.5 Kelvin Model 55
5.6 Burgers or Four-element Model 57
5.7 Generalized Maxwell and Kelvin Models 64
5.8 Retardation Spectrum for t^n 69
5.9 Differential Form of Constitutive Equations for Simple Stress States 71
5.10 Differential Form of Constitutive Equations for Multiaxial Stress States 78
5.11 Integral Representation of Viscoelastic Constitutive Equations 81
5.12 Creep Compliance 81
5.13 Relaxation Modulus 81
5.14 Boltzmann's Superposition Principle and Integral Representation 82
5.15 Relation Between Creep Compliance and Relaxation Modulus 85
5.16 Generalization of the Integral Representation to Three-Dimensions 87
5.17 Behavior of Linear Viscoelastic Material under Oscillating Loading 89
5.18 Complex Modulus and Compliance 90
5.19 Dissipation 95
5.20 Complex Compliance and Complex Modulus of Some Viscoelastic Models 96
5.21 Maxwell Model 96
5.22 Kelvin Model 97
5.23 Burgers Model 99
5.24 Relation Between the Relaxation Modulus and the Complex Relaxation Modulus 100
5.25 Relation Between Creep Compliance and Complex Compliance 103
5.26 Complex Compliance for t^n 104
5.27 Temperature Effect and Time-Temperature Superposition Principle 105

CHAPTER 6. LINEAR VISCOELASTIC STRESS ANALYSIS 108
6.1 Introduction 108
6.2 Beam Problems 109
6.3 Stress Analysis of Quasi-static Viscoelastic Problems Using the Elastic-Viscoelastic Correspondence Principle 119
6.4 Thick-walled Viscoelastic Tube 122
6.5 Point Force Acting on the Surface of a Semi-infinite Viscoelastic Solid 128
6.6 Concluding Remarks 130

CHAPTER 7. MULTIPLE INTEGRAL REPRESENTATION 131
7.1 Introduction 131
7.2 Nonlinear Viscoelastic Behavior under Uniaxial Loading 132
7.3 Nonlinear Viscoelastic Behavior under Multiaxial Stress State 137
7.4 A Linearly Compressible Material 140
7.5 Incompressible Material Assumption 144
7.6 Linearly Compressible, II 150
7.7 Constant Volume 151
7.8 Incompressible and Linearly Compressible Creep in Terms of σ 151
7.9 Incompressible and Linearly Compressible Relaxation in Terms of ε 153
7.10 Constitutive Relations under Biaxial Stress and Strain 153
7.11 Constitutive Relations under Uniaxial Stress and Strain 156
7.12 Strain Components for Biaxial and Uniaxial Stress States, Compressible Material 157
7.13 Strain Components for Biaxial and Uniaxial Stress States, Linearly Compressible Material 161
7.14 Stress Components for Biaxial and Uniaxial Strain States 163
7.15 Approximating Nonlinear Constitutive Equations under Short Time Loading 165
7.16 Superposed Small Loading on a Large Constant Loading 168
7.17 Other Representations 169
7.18 Finite Linear Viscoelasticity 169
7.19 Elastic Fluid Theory 171
7.20 Thermodynamic Constitutive Theory 172

CHAPTER 8. NONLINEAR CREEP AT CONSTANT STRESS AND RELAXATION AT CONSTANT STRAIN 176
8.1 Introduction 176
8.2 Constitutive Equations for 3×3 Matrix 176
8.3 Components of Strain for Creep at Constant Stress 177
8.4 Components of Stress for Relaxation at Constant Strain 182
8.5 Biaxial Constitutive Equations for 2×2 Matrix 185
8.6 Components of Strain (or Stress) for Biaxial States for 2×2 Matrix 186
8.7 Constitutive Equations for Linearly Compressible Material 187
8.8 Components of Strain for Creep of Linearly Compressible Material 187
8.9 Components of Stress for Relaxation of Linearly Compressible Material 189
8.10 Poisson's Ratio 189
8.11 Time Functions 190
8.12 Determination of Kernel Functions for Constant Stress Creep 198
8.13 Determination of Kernel Functions for Constant-Strain Stress-Relaxation 205
8.14 Experimental Results of Creep 208

CHAPTER 9. NONLINEAR CREEP (OR RELAXATION) UNDER VARIABLE STRESS (OR STRAIN) 220
9.1 Introduction 220
9.2 Direct Determination of Kernel Functions 220
9.3 Product-Form Approximation of Kernel Functions 227
9.4 Additive Forms of Approximation of Kernel Functions 228
9.5 Modified Superposition Method 229

9.6 Physical Linearity Approximation of Kernel Functions 233
9.7 Comparison 235

CHAPTER 10. CONVERSION AND MIXING OF NONLINEAR CREEP AND RELAXATION 239
10.1 Introduction 239
10.2 Relation Between Creep and Stress Relaxation for Uniaxial Nonlinear Viscoelasticity 240
10.3 Example: Prediction of Uniaxial Stress Relaxation from Creep of Nonlinear Viscoelastic Material 242
10.4 Relation Between Creep and Relaxation for Biaxial Nonlinear Viscoelasticity 244
10.5 Behavior of Nonlinear Viscoelastic Material under Simultaneous Stress Relaxation in Tension and Creep in Torsion 245
10.6 Prediction of Creep and Relaxation under Arbitrary Input 247

CHAPTER 11. EFFECT OF TEMPERATURE ON NONLINEAR VISCOELASTIC MATERIALS 249
11.1 Introduction 249
11.2 Nonlinear Creep Behavior at Elevated Temperatures 249
11.3 Determination of Temperature Dependent Kernel Functions 251
11.4 Creep Behavior under Continuously Varying Temperature–Uniaxial Case 256
11.5 Creep Behavior under Continuously Varying Temperature for Combined Tension and Torsion 263
11.6 Thermal Expansion Instability 264

CHAPTER 12. NONLINEAR VISCOELASTIC STRESS ANALYSIS 268
12.1 Introduction 268
12.2 Solid Circular Cross-section Shaft under Twisting 268
12.3 Beam under Pure Bending 273
12.4 Thick-walled Cylinder under Axially Symmetric Loading 279

CHAPTER 13. EXPERIMENTAL METHODS 289
13.1 Introduction 289
13.2 Loading Apparatus for Creep 289
13.3 Load Application 291
13.4 Test Specimen 291
13.5 Uniform Stressing or Straining 292
13.6 Strain Measurement 294
13.7 Temperature Control 295
13.8 Humidity and Temperature Controlled Room 296
13.9 Internal Pressure 297
13.10 Strain Control and Stress Measurement for Relaxation 298
13.11 A Machine for Combined Tension and Torsion 299

APPENDIX A1. LIST OF SYMBOLS 304

APPENDIX A2. MATHEMATICAL DESCRIPTION OF NONLINEAR VISCOELASTIC CONSTITUTIVE RELATION 312
A2.1 Introduction 312
A2.2 Material Properties 312

A2.3 Multiple Integral Representation of Initially Isotropic Materials (Relaxation Form) 313
A2.4 The Inverse Relation (Creep Form) 316

APPENDIX A3. UNIT STEP FUNCTION AND UNIT IMPULSE FUNCTION 317
A3.1 Unit Step Function or Heaviside Unit Function 317
A3.2 Signum Function 319
A3.3 Unit Impulse or Dirac Delta Function 320
A3.4 Relation Between Unit Step Function and Unit Impulse Function 322
A3.5 Dirac Delta Function or Heaviside Function in Evaluation of Integrals 323

APPENDIX A4. LAPLACE TRANSFORMATION 326
A4.1 Definition of the Laplace Transformation 326
A4.2 Sufficient Conditions for Existence of Laplace Transforms 326
A4.3 Some Important Properties of Laplace Transforms 327
A4.4 The Inverse Laplace Transform 333
A4.5 Partial Fraction Expansion 333
A4.6 Some Uses of the Laplace Transform 334

APPENDIX A5. DERIVATION OF THE MODIFIED SUPERPOSITION PRINCIPLE FROM THE MULTIPLE INTEGRAL REPRESENTATION 336
A5.1 Second Order Term 336
A5.2 Third Order Term 338
A5.3 Application to Third Order Multiple Integrals for Creep 339

APPENDIX A6. CONVERSION TABLES 341

BIBLIOGRAPHY 344

SUBJECT INDEX 361

AUTHOR INDEX 365