Preface

There are many approaches to noncommutative geometry and its use in physics, the operator algebra and C^*-algebra one, the deformation quantization one, the quantum group one, and the matrix algebra/fuzzy geometry one. This volume introduces and develops the subject by presenting in particular the ideas and methods recently pursued by Julius Wess and his group.

These methods combine the deformation quantization approach based on the notion of star product and the deformed (quantum) symmetries methods based on the theory of quantum groups. The merging of these two techniques has proven very fruitful in order to formulate field theories on noncommutative spaces. The aim of the book is to give an introduction to these topics and to prepare the reader to enter the research field himself/herself. This has developed from the constant interest of Prof. W. Beiglboeck, editor of LNP, in this project, and from the authors experience in conferences and schools on the subject, especially from their interaction with students and young researchers.

In fact quite a few chapters in the book were written with a double purpose, on the one hand as contributions for school or conference proceedings and on the other hand as chapters for the present book. These are now harmonized and complemented by a couple of contributions that have been written to provide a wider background, to widen the scope, and to underline the power of our methods.

The different chapters however remain essentially self-consistent and can be read independently. Subject to the individual interests of the reader they can be grouped by topic: noncommutative gauge theory (Chaps. 1, 2, 4, 5), noncommutative gravity (Chaps. 1, 3, 8), and noncommutative geometry and quantum groups (Chaps. 6, 7, 9). This very structure of the book took definite shape a little more than a year ago, at the Alessandria conference "Noncommutative Spacetime Geometries" in March 2007, where all the authors met. At the Bayrishzell workshop "On Noncommutativity and Physics" in May 2007 the order of the chapters was then finalized.

The order of the chapters is "physics first"; the mathematics follows the physical motivations in order to strengthen the physical intuition and investigations and to provide a sharpening of the mathematical methods. These is turn are then used for further physical developments. Accordingly the book is divided into a more physical first part and a more mathematical second part, although the division is not sharp, physical applications being considered in the second part too.

The first chapter is an introduction and an overview. The reader encounters the notion of star product and is introduced to the differential calculus on noncommutative spaces and to the deformed Lie algebras (twisted Hopf algebras) of gauge transformations and diffeomorphisms. The second chapter develops in more detail deformed gauge theories. Pedagogic examples with matter fields are also presented. The third chapter discusses in the same spirit the deformed algebra of differential operators and hence a deformation of the theory of gravity. Changes to the original text of Julius Wess mainly appear in the added footnotes and in the added Appendix 1.9.

The fourth chapter is a comparison between two approaches to noncommutative gauge theory, the twisted gauge theory approach (based on deformed Lie algebras) and the Seiberg–Witten approach.

Field theories can be studied also on more general noncommutative spaces, not just on the Moyal–Weyl one characterized by the $x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}$ noncommutative relations among coordinates (with $\theta^{\mu\nu}$ constant). Chapter 5 describes the case of κ-deformed spacetime.

Part II of the book opens with a chapter on the basics of noncommutative manifolds in the C^*-algebraic approach, the guiding example being the quantum mechanical phase space, i.e., the Moyal–Weyl noncommutative space. Quantum groups (noncommutative manifolds with a group structure) are then studied in Chap. 7. Their quantum Lie algebras are also studied, quantum Lie algebras being the underlying symmetries of field theories on noncommutative spaces. Chapter 8 complements Chap. 3 and studies noncommutative geometries obtained by deforming commutative geometries via a twist. These geometries have twisted symmetries (twisted quantum group symmetries). Twisted diffeomorphisms lead to a noncommutative theory of gravity.

While twisting of spacetime symmetries leads to deformed field theories, twisting of dynamical symmetries leads to new (deformed) quantum integrable systems. The last chapter deals with this other application of twisted symmetries. In a sense this chapter closes a circle, we deform field theories by considering noncommutative spacetimes. These are obtained via a twist procedure. We recognize and exploit the underlying twisted and quantum group symmetries. These structures first occurred in $1+1$-dimensional quantum integrable systems; the twist procedure can be also applied in this context and leads to new physical systems.

A final chapter has later been added and describes the contributions of Julius Wess to noncommutative geometry. As can be inferred from his joint works he was able to enroll many students and collaborators in his research projects. This was due to his scientific charisma, always downplayed, and to the easiness in relating with colleagues and younger collaborators, a characteristic aspect of his personality.

Julius Wess was extremely active until his last day, his constant passion for research was so strongly conveyed that concentration and energy for advancing in the research were multiplied. In his vision the main aims and questions were always in the foreground, progress was constant, in many little steps, like that patient walking pace you keep when aiming at the very top. We miss his encouragement, hints, and judgments and that very state of searching together that empowered our discovering abilities. We hope the reader can experience his calm impetus along with the formulae in this book, and thus be more easily brought to the research frontiers of this field to be further developed.

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