

Contents

1	Preliminaries	1
1.1	Reflections and Roots	1
1.1.1	Rank One Endomorphisms	1
1.1.2	Projections, Transvections, Reflections	2
1.1.3	Reflections	2
1.1.4	Commuting Reflections	3
1.2	Reflection Groups	4
1.2.1	Orthogonal Decomposition	5
1.2.2	The Shephard–Todd Classification	6
1.2.3	Reflecting Pairs	8
2	Prerequisites and Complements in Commutative Algebra ..	11
2.1	Finite Ring Extensions	11
2.1.1	Properties and Definitions	11
2.1.2	Spectra and Finite Extensions	12
2.1.3	Case of Integrally Closed Rings	13
2.1.4	Krull Dimension: First Definitions	13
2.2	Jacobson Rings and Hilbert’s Nullstellensatz	15
2.2.1	On Maximal Ideal of Polynomial Algebras	15
2.2.2	Radicals and Jacobson Rings, Application to Algebraic Varieties	19
2.3	Graded Algebras and Modules	20
2.3.1	Graded Modules	20
2.3.2	Elementary Constructions	21
2.3.3	Koszul Complex	22
2.3.4	Graded Algebras and Modules	23
2.3.5	The Hilbert–Serre Theorem	23
2.3.6	Nakayama’s Lemma	24
2.4	Polynomial Algebras and Parameters Subalgebras	26
2.4.1	Degrees and Jacobian	26
2.4.2	Systems of Parameters	28
2.4.3	The Chevalley Theorem	31

3	Polynomial Invariants of Finite Linear Groups	35
3.1	Finite Groups Invariants	35
3.1.1	Generalities	35
3.1.2	Case of Height One Primes	37
3.2	Finite Linear Groups on Symmetric Algebras	38
3.2.1	Ramification and Reflecting Pairs.....	39
3.2.2	Linear Characters Associated with Reflecting Hyperplanes	40
3.3	Coinvariant Algebra and Harmonic Polynomials	44
3.3.1	The Coinvariant Algebra	44
3.3.2	Galois Twisting of a Representation	45
3.3.3	Differential Operators, Harmonic Polynomials	46
3.4	Graded Characters and Applications.....	48
3.4.1	Graded Characters of Graded kG -Modules	49
3.4.2	Isotypic Components of the Symmetric Algebra	50
3.4.3	Some Numerical Identities	50
3.4.4	Isotypic Components Are Cohen-Macaulay	52
3.4.5	Computations with Power Series.....	52
3.4.6	A Simple Example	55
4	Finite Reflection Groups in Characteristic Zero	57
4.1	The Shephard-Todd/Chevalley-Serre Theorem	57
4.2	Steinberg Theorem and First Applications	60
4.2.1	The Jacobian as a Monomial	60
4.2.2	Action of the Normalizer and Generalized Degrees	60
4.2.3	Steinberg Theorem	62
4.2.4	Fixed Points of Elements of G	64
4.2.5	Braid Groups	65
4.3	Coinvariant Algebra and Harmonic Polynomials	67
4.3.1	On the Coinvariant Algebra	67
4.3.2	Linear Characters and Their Associated Polynomials ..	69
4.3.3	The Harmonic Elements of a Reflection Group and the Poincaré Duality	72
4.4	Application to Braid Groups.....	75
4.4.1	Discriminants and Length	75
4.4.2	Complement: Artin-Like Presentations of the Braid Diagrams	77
4.5	Graded Multiplicities and Solomon's Theorem	78
4.5.1	Preliminary: Graded Dimension of $(S \otimes V)^G$	78
4.5.2	Exponents and Gutkin-Opdam Matrices.....	80
4.5.3	Solomon Theorem	89
4.5.4	Derivations and Differential Forms on V	91
4.5.5	First Applications of Solomon's Theorem	94

5 Eigenspaces and Regular Elements	97
5.1 Eigenspaces.....	97
5.1.1 Pianzola–Weiss Formula	97
5.1.2 Maximal Eigenspaces: Lehrer–Springer Theory	99
5.2 Regular Elements	104
5.2.1 First Properties.....	104
5.2.2 Exponents and Eigenvalues of Regular Elements	106
5.3 Regular Braid Automorphisms	112
5.3.1 Lifting Regular Automorphisms: Case When the Base Point Is an Eigenvector	112
5.3.2 Lifting Regular Automorphisms: General Case	115
5.3.3 Lifting Springer Theory.....	117
 Coxeter and Artin–Like Presentations	 119
 A Coxeter and Artin–Like Presentations	 119
A.1 Meaning of the Diagrams	119
A.1.1 Diagrams for the Reflection Groups	119
A.1.2 Braid Diagrams.....	121
A.2 Tables.....	124
 References	 133
 Index	 137