Contents

1 What Is NONLINEAR? 1
1.1 Nature and Science 1
1.1.1 *Natura Vexata* 1
1.1.2 Syndrome 4
1.1.3 *Déconstruction* of Linear Theory 5
1.2 The Scale of Phenomenon / Theory with Scale 6
1.2.1 The Role of *Scale* in Scientific Revolutions 6
1.2.2 The Mathematical Recognition of *Scale* 8
1.3 The Territory of Linear Theory 10
1.3.1 Linear Space —The Horizon of Mathematical Science 10
1.3.2 The Mathematical Definition of Vectors 12
1.3.3 Graphs—Geometric Representation of Laws 15
1.3.4 Exponential Law 20
1.4 Nonlinearity—Phenomenology and Structures 24
1.4.1 Nonlinear Phenomena 24
1.4.2 The Typology of Distortion 25
1.4.3 Nonlinearity Emerging in Small Scale—*Singularity* 27
1.4.4 Nonlinearity Escaping from Linearity—*Criticality* 29
1.4.5 Bifurcation (Polyvalency) and Discontinuity 31
Notes 33
Problems 41
Solutions 41
References 43

2 From Cosmos to Chaos 45
2.1 The Order of Nature—A Geometric View 45
2.1.1 Galileo's Natural Philosophy 45
2.1.2 Geometric Description of Events 46
2.1.3 Universality Discovered by Newton 48
2.2 Function—The Mathematical Representation of Order 52
2.2.1 Motion and Function 52
2.2.2 Nonlinear Regime 54

Bibliografische Informationen
http://d-nb.info/995338310

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

2.2.3 Beyond the Functional Representation of Motion 56
2.3 Decomposition—Elucidation of Order 58
2.3.1 The Mathematical Representation of Causality 58
2.3.2 Exponential Law—A Basic Form of *Group* 60
2.3.3 *Resonance*—Undecomposable Motion 62
2.3.4 Nonlinear Dynamics—An Infinite Chain of Interacting Modes 65
2.3.5 Chaos—Motion in the Infinite Period 67
2.3.6 Separability/Inseparability 69
2.4 Invariance in Dynamics 74
2.4.1 Constants of Motion 74
2.4.2 *Chaos*—True Evolution 80
2.4.3 Collective Order 81
2.4.4 Complete Solution—The Frame of Space Embodying Order 83
2.4.5 The Difficulty of *Infinity* 85
2.5 Symmetry and Conservation Law 86
2.5.1 Symmetry in Dynamical System 86
2.5.2 The Deep Structure of Dynamical System 87
2.5.3 The Translation of Motion and Non-motion 91
2.5.4 Chaos—The Impossibility of Decomposition 94
Notes 97
Problems 105
Solutions 106
References 109

3 The Challenge of Macro-Systems 111
3.1 The Difficulty of Prediction 111
3.1.1 Chaos in Phenomenological Recognition 111
3.1.2 Stability 112
3.1.3 Attractors 116
3.1.4 Stability and Integrability 119
3.2 Randomness as Hypothetical Simplicity 121
3.2.1 Stochastic Process 121
3.2.2 Representation of Motion by Transition Probability 123
3.2.3 H-Theorem 125
3.2.4 Statistical Equilibrium 127
3.2.5 Statistically Plausible Particular Solutions 131
3.3 Collective Phenomena 132
3.3.1 Nonequilibrium and Macroscopic Dynamics 132
3.3.2 A Model of Collective Motion 133
3.3.3 A Statistical Model of Collisions 137
Notes 140
Problems 147

Solutions 148
References 150

4 Interactions of Micro and Macro Hierarchies 153
4.1 Structure and Scale Hierarchy 153
4.1.1 Crossing-Over Hierarchies 153
4.1.2 Connection of Scale Hierarchies—Structure 154
4.2 Topology—A System of Differences 157
4.2.1 The Topology of Geometry 157
4.2.2 Scale Hierarchy and Topology 158
4.2.3 Fractals—Aggregates of Scales 159
4.3 The Scale of Event / The Scale of Law 161
4.3.1 Scaling and Representation 161
4.3.2 Scale Separation 164
4.3.3 Spontaneous Selection of Scale by Nonlinearity 167
4.3.4 Singularity—Ideal Limit of Scale-Invariant Structure 168
4.4 Connections of Scale Hierarchies 170
4.4.1 Complexity—Structures with Multiple Aspects 170
4.4.2 Singular Perturbation 171
4.4.3 Collaborations of Nonlinearity and Singular Perturbation 174
4.4.4 Localized Structures in Space–Time 178
4.4.5 Irreducible Couplings of Multi-Scales 183
Notes 187
Problems 201
Solutions 202
References 206

Index 209