Contents

List of Figures *xiii*
List of Tables *xv*
Preface *xvii*

1 The Field of Dynamic Climatology 1

2 Mathematics 8
2.1 Geometry 9
2.1.1 The Circle, Sphere, and Pi 9
2.1.2 The Ellipse 10
2.2 Differential Calculus 12
2.3 Partial Derivatives 16
2.4 Integral Calculus 19
2.5 Development of Calculus 22
2.6 Vectors 26
2.6.1 Addition and Subtraction 27
2.6.2 Multiplication 27
2.6.3 Differentiation 30
2.6.4 Differential Operator 32
2.6.5 Tensors 34
2.7 The Exponential and Complex Numbers 34
2.7.1 The Number e 34
2.7.2 The Imaginary Number 36
2.7.3 Complex Numbers 37
2.7.4 Cosines and Sines as Complex Numbers 38

2.8 Finite Differences 40
2.8.1 Development 40
2.8.2 Example 41
2.9 Comment 44

3 Statistics **47**
3.1 Data 47
3.2 One Variable Descriptive Statistics 51
3.3 Two Variables 58
3.3.1 Covariance 58
3.3.2 Regression 58
3.3.3 Relationship Variables 62
3.4 Dependence 63
3.4.1 Auto-covariance 63
3.4.2 Fourier Series 64
3.4.3 Complex Representation 69
3.4.4 Fourier Transform and Convolution 69
3.4.5 Spectral Analysis of Non-periodic Functions 71
3.4.6 Non-normal and Dependent Data 75
3.4.7 Dependence Summary 76
3.5 Dependence for More Than One Variable 76
3.5.1 Two Entities 76
3.5.2 The Transfer Function 78
3.5.3 Three Entities 78
3.6 Comment 79

4 Mechanics **80**
4.1 Newton's Definitions and Laws 80
4.2 Base Units 82
4.3 Derived Units 83
4.3.1 Speed 83
4.3.2 Acceleration 83
4.3.3 Force 83
4.3.4 Quantity of Motion 89
4.3.5 Energy 90
4.3.6 Power 96
4.3.7 Force and Momentum of Curved Motion for Solid Bodies 97
4.4 Discussion 101

5 Thermodynamics **102**
5.1 Definitions 102

5.2 The Equation of State – the Macroscopic Approach 103
5.2.1 Temperature 103
5.2.2 Pressure 104
5.2.3 Pressure and Volume – Boyle's or Mariotte's Law 104
5.2.4 Gay-Lussac's Law 105
5.2.5 Dalton's Law 105
5.2.6 Avogadro's Hypothesis 106
5.2.7 Equation of State 107
5.2.8 The Equation of State – the Microscopic Approach 109
5.3 Atmospheric Composition 111
5.4 Heat 113
5.4.1 General Comments on Temperature and Heat 113
5.4.2 Early Development of the Concept of Heat 114
5.5 The First Law of Thermodynamics 117
5.5.1 Clausius's Statement 117
5.5.2 Recent Statement 118
5.5.3 Specific Heat Capacities and the Gas Constant 119
5.5.4 Standard Forms of the First Law 120
5.6 The Carnot Cycle 120
5.7 Dry Adiabats and Potential Temperature 122
5.8 The Second Law of Thermodynamics 124
5.9 Water 127
5.9.1 Latent Heats 127
5.9.2 Expressions for Humidity 128
5.9.3 Equation of State for Moist Air 131
5.9.4 Water and Adiabats 132
5.10 Discussion 133

6 Radiation **134**
6.1 Early Work 134
6.1.1 Light 134
6.1.2 Extension to Other Wavelengths 135
6.2 Quanta 135
6.3 Definitions and Laws of Radiation 138
6.3.1 Bouger's Law 138
6.3.2 Lambert's Law 138
6.3.3 Intensity 139
6.3.4 Flux 140

6.3.5 Special Cases 141
6.3.6 Kirchhoff's Law 142
6.3.7 Stefan–Boltzmann's Law 143
6.3.8 Wien's Law 144
6.3.9 Planck's Equation 145
6.3.10 Schwarzschild's Equation 147
6.3.11 Scattering 148
6.4 Application to the Earth 149
6.4.1 Solar Receipt 149
6.4.2 Earth–Sun–Space Equilibrium 153
6.4.3 Trigonometry of Solar Radiation 154
6.4.4 Atmospheric Gases 157
6.5 Comment 162

7 Atmospheric Equations **163**
7.1 The Nature of Fluids 163
7.2 Continuity – Conservation of Mass 164
7.3 Molecular Viscosity 165
7.3.1 Newtonian Approach 166
7.3.2 Statistical Approach 167
7.4 The Stress Tensor 169
7.5 Navier–Stokes Equations 170
7.6 Turbulent Eddy Viscosity 171
7.6.1 Reynolds' Stresses 172
7.6.2 Molecular Analogy 173
7.6.3 Extension to Other Fluxes 174
7.6.4 Richardson Number 175
7.7 The Vector Equation of Motion 175
7.7.1 Relative Acceleration 175
7.7.2 The Complete Vector Equation 176
7.8 General Coordinates 176
7.8.1 Cartesian Coordinates 177
7.8.2 Spherical Polar Coordinates 177
7.8.3 Natural Coordinates 182
7.8.4 Vertical Coordinate 183
7.9 Some Simple Solutions 184
7.9.1 Synoptic Scale Approximations 185
7.10 Fluid Rotation 188
7.11 The Equation Set 193
7.12 Comment 194

8 Observed Angular Momentum and Energy **195**
8.1 Perspective 195

8.2 Angular Momentum 196
8.2.1 Total Angular Momentum 196
8.2.2 Angular Momentum Transfer 196
8.2.3 Estimates 199
8.3 The Partition of Energy 201
8.4 The Lorenz Model of Energy Flow 203
8.5 Heat Budget 206
8.6 Water Budget 209
8.7 Conversion Between Scales of Motion 209
8.8 The General Circulation 210

9 Towards an Explanation of Climate **212**
9.1 The Problem 212
9.1.1 Specification 212
9.1.2 The Method 215
9.1.3 Requirements 216
9.2 Numerical Modeling 218
9.2.1 Early Development 218
9.2.2 The Electronic Computer 219
9.3 Climate Modeling 221

10 Concluding Remarks **223**

Appendix A: Power Notation *225*
Appendix B: Constants *226*
Appendix C: Conversions *228*
Appendix D: World Data *230*
Bibliography *232*
Index *247*