
Contents

1	Definition of stochastic process. Cylinder σ-algebra, finite-dimensional distributions, the Kolmogorov theorem	1
	Theoretical grounds	1
	Bibliography	3
	Problems	3
	Hints	7
	Answers and Solutions	9
2	Characteristics of a stochastic process. Mean and covariance functions. Characteristic functions	11
	Theoretical grounds	11
	Bibliography	13
	Problems	13
	Hints	16
	Answers and Solutions	17
3	Trajectories. Modifications. Filtrations	21
	Theoretical grounds	21
	Bibliography	24
	Problems	24
	Hints	29
	Answers and Solutions	31
4	Continuity. Differentiability. Integrability	33
	Theoretical grounds	33
	Bibliography	34
	Problems	34
	Hints	38
	Answers and Solutions	40

5	Stochastic processes with independent increments. Wiener and Poisson processes. Poisson point measures	43
	Theoretical grounds	43
	Bibliography	46
	Problems	47
	Hints	53
	Answers and Solutions	54
6	Gaussian processes	59
	Theoretical grounds	59
	Bibliography	61
	Problems	62
	Hints	66
	Answers and Solutions	67
7	Martingales and related processes in discrete and continuous time. Stopping times	71
	Theoretical grounds	71
	Bibliography	79
	Problems	79
	Hints	93
	Answers and Solutions	98
8	Stationary discrete- and continuous-time processes. Stochastic integral over measure with orthogonal values	107
	Theoretical grounds	107
	Bibliography	110
	Problems	111
	Hints	119
	Answers and Solutions	122
9	Prediction and interpolation	129
	Theoretical grounds	129
	Bibliography	130
	Problems	131
	Hints	133
	Answers and Solutions	135
10	Markov chains: Discrete and continuous time	137
	Theoretical grounds	137
	Bibliography	140
	Problems	140
	Hints	152
	Answers and Solutions	154

11	Renewal theory. Queuing theory	159
	Theoretical grounds	159
	Bibliography	162
	Problems	162
	Hints	169
	Answers and Solutions	170
12	Markov and diffusion processes	175
	Theoretical grounds	175
	Bibliography	182
	Problems	182
	Hints	186
	Answers and Solutions	188
13	Itô stochastic integral. Itô formula. Tanaka formula	193
	Theoretical grounds	193
	Bibliography	196
	Problems	196
	Hints	205
	Answers and Solutions	209
14	Stochastic differential equations	215
	Theoretical grounds	215
	Bibliography	217
	Problems	217
	Hints	223
	Answers and Solutions	225
15	Optimal stopping of random sequences and processes	229
	Theoretical grounds	229
	Bibliography	231
	Problems	231
	Hints	235
	Answers and Solutions	237
16	Measures in a functional spaces. Weak convergence, probability metrics. Functional limit theorems	241
	Theoretical grounds	241
	Bibliography	250
	Problems	250
	Hints	259
	Answers and Solutions	262

17	Statistics of stochastic processes	271
	Theoretical grounds	271
	Bibliography	281
	Problems	281
	Hints	286
	Answers and Solutions	287
18	Stochastic processes in financial mathematics (discrete time)	303
	Theoretical grounds	303
	Bibliography	306
	Problems	306
	Hints	310
	Answers and Solutions	311
19	Stochastic processes in financial mathematics (continuous time)	315
	Theoretical grounds	315
	Bibliography	317
	Problems	317
	Hints	322
	Answers and Solutions	322
20	Basic functionals of the risk theory	327
	Theoretical grounds	327
	Bibliography	343
	Problems	343
	Hints	348
	Answers and Solutions	350
Appendix		359
	List of abbreviations	364
	List of probability distributions	364
	List of symbols	365
	References	367
Index		371