# Contents

### Preface

Part 1	The context	
Chapte	1 The structure of the argument	2
Chapte	r 2 Examples of population models	4
2.1	Introduction	
2.2	The closed-system cohort survival model	
2.3	The open-system single-region cohort survival model	
2.4	The multi-region cohort survival model	
2.5	The problem of rate measurement	
Part 2	Accounts and models for multi-region aggregate populations	
Chapter	3 Principles of accounting	18
3.1	Introduction	
3.2	Accounts	
3.3	Basic population accounts	
3.4	Use of accounting equations	
3.5	Summary of results for the standard case	
Chapter	4 'At-risk' populations	29
4.1	The concept, and a basic assumption	
4.2	At-risk populations for births	
4.3	At-risk populations for deaths	
4.4	At-risk populations for migration	
4.5	The use of at-risk population concepts: preliminary discussion	
Chapter	5 An account-based model for the aggregate case	36
5.1	Procedures for estimating unknown minor flows from known data. 1: the standard case	
5.2	A note on surviving infant migrants	
5.3	Procedures for estimating unknown minor flows from known data. 2: the	
	non-standard cases	
5.4	An account-based model	
5.5	Uses of the account-based model: historical analyses and future projections	
5.6	A note on appropriate non-iterative forms of account-based model	
Chapter	6 The transition rate form of the model	45
6.1	Basic principles	
6.2	Application to the demographic model	

6.3 Some comments

#### vi CONTENTS

#### Chapter 7 An example

- 7.1 Introduction
- 7.2 Regions
- 7.3 The population accounts table in conceptual form
- 7.4 Data availability and the time period adopted
- 7.5 Estimation of known flows and stocks from data
- 7.6 Initial populations
- 7.7 Deaths
- 7.8 Births
- 7.9 'Migration and survival' flows
- 7.10 'Birth and migration' flows
- 7.11 Concluding comments on the estimation of flows and stocks
- 7.12 A worked example of the account-based model for an historical period
- 7.13 Projection using the transition rate model
- 7.14 Concluding comments

#### Part 3 Accounts and models for multi-region age-sex disaggregated populations

Chapter	8 Principles of age and sex disaggregation	88
8.1	Introduction: notational developments	
8.2	Ageing	
8.3	Structure of Part 3	
Chapter	9 Age-sex disaggregated accounts	93
9.1	Outline of the accounts for the general case	
9.2	A preliminary classification of age-group types	
9.3	The accounting equations	
9.4	Coefficients which relate variables within submatrices	
9.5	Revision of accounting equations	
9.6	An alternative presentation of the accounting equations	
9.7	Accounting equations for the simple case	
9.8	A note on sex	
Chapter	10 Age sex disaggregated at risk populations	111
	Introductions the at rick populations needed	111
10.1	Principles of coloulation and main recults	
10.2	At rich annulations in the simple cost	
10.5	At-risk populations in the simple case	
Chapter	11 An account-based model for age-sex disaggregated populations	114
11.1	Estimation of unknown minor flows	
11.2	A note on migrating, surviving infants	
11.3	An account-based model	
11.4	The account-based model used for projection	
11.5	The account-based model for the simple case	

- 11.6 Disaggregation by sex
- 11.7 Possible approximate non-iterative forms of the model

	CONTENTS	vii
Chapter 12 The transition rate form of the models in the disaggregated case, and connections to the Rogers and Stone models 11		
12.1	The general case	
12.2	The simple case	
12.3	Disaggregation by sex	
12.4	Comparison with the Rogers model	
12.5	Comparison with Stone's accounts and models	
12.6	Concluding comments	
Chapter	13 Examples	158
13.1	Introduction	
13.2	A set of age-disaggregated accounts: example 3	
13.3	The structure of a set of expanded age-disaggregated accounts	
13.4	The account-based model applied to Middle Earth	
13.5	Rates associated with accounts tables	
13.6	Projection models	
13.7	Example 4: brief notes on a real-world, simple case example for the West	
10.0	Riding of Yorkshire	
13.8	Example 5: brief notes on a real-world, general case example for East and	
12.0	West Hunslet	
13.9	Concluding comments	
Part 4	Multi-region account-based life tables	
Chapter	14 Life-table accounts	216
14.1	Introduction	
14.2	A review of rate definitions, and the introduction of life-table rates	
14.3	Life-table accounts	
14.4	Connections to conventional life-table rates	
Chapter	15 A life-table account-based model	230
15.1	Summary of the main steps	
15.2	The model	
Chapter	16 Multi-regional account-based life tables	237
16.1	The closed-system life table	
16.2	The multi-regional exact-age life table	
16.3	Models associated with the life table	
16.4	The connections between life-table accounts and age-group accounts	
16.5	Age group life tables	
10.0	Concluding comments	

# Part 5 Conclusions

Chapter 17 A summary of methods, potential applications and connections to other and ongoing work

#### viii contents

- 17.1 Summary of methods introduced17.2 Potential applications17.3 Connections to other work

- 17.4 Connections to ongoing work

## Appendixes

Appendi	ix 1 The estimation of migration flows for the aggregate case example	282
A1.1	'Migration and survival' flows	
A1.2	'Birth and migration' flows	
Appendi	ix 2 Calculation of $p_{rs}$ , $q_{rs}$ , $c_{rs}$ and $\pi_s$ coefficients	295
A2.1	Survival coefficients	
A2.2	Death coefficients	
A2.3	Birth coefficients	
Appendi	ix 3 Use of aggregation and disaggregation matrix operators	306
A3.1	Definitions	
A3.2	Survival coefficients	
A3.3	Death coefficients	
Appendi	ix 4 Age-sex disaggregated at-risk populations: detailed calculations	311
A4.1	Introduction	
A4.2	The main accounts at-risk population for births	
A4.3	At-risk population for deaths	
A4.4	The birth accounts at-risk population for births	
A4.5	Populations at risk in the simple case	
A4.6	Summary	
A4.7	Disaggregation by sex	
Append	ix 5 The calculation of $\theta$ and $\psi$ coefficients	332
Append	ix 6 Models with time as a continuous variable	336
Append	ix 7 Calculation of a set of expanded birth accounts and the birth section of	E
	the accounts for the multi-regional cohort survival model	338
Referen	ces and further reading	347
Index		351