

CONTENTS

Preface	xi
0.1 Short Introduction	xi
0.2 Introduction	xii
0.2.1 Background	xii
0.2.2 General Approach of the Text	xiii
0.2.2.1 Motivational Parts	xiv
0.2.2.2 Core Theoretical Parts	xv
0.2.2.3 Parts Containing Examples and Problems	xvi
0.2.3 Miscellaneous Aspects of the Approach	xvii
0.2.4 Requirements of Applications	xviii
0.2.5 Remarks on the Role of Abstraction in Natural Sciences	xix
Acknowledgments	xxi
1 Calculus I	1
1.1 A Sketch of the Development of Rigor in Calculus and Analysis	1
1.2 Basics	3
1.2.1 Elementary Mathematical Logic	3
Problems	10
	vii

1.2.2	Sets	12
	Problems	19
1.2.3	Maps	20
	Problems	32
1.3	Limits and Continuous Functions	35
1.3.1	Limits of Real-Number Sequences	35
	Problems	56
1.3.2	Continuous Functions	58
	Problems	81
1.4	Differentiation	83
	Problems	99
1.5	Applications of Differentiation	101
	Problems	155
1.6	Riemann Integration	159
	Problems	186
2	Calculus II	191
2.1	Techniques of Integration	191
2.1.1	Change of Variables	191
	Problems	204
2.1.2	Integration by Parts	205
	Problems	216
2.1.3	Partial Fractions	217
	Problems	229
2.1.4	Approximate Numerical Calculation of Integrals	230
	Problems	237
2.2	Improper Integrals	238
	Problems	258
2.3	Series of Real Numbers	261
	Problems	292
2.4	Series of Functions	293
	Problems	338
2.5	Analytical Geometry and Elementary Vector Calculus	343
2.5.1	Metric Spaces	344
	Problems	351
2.5.2	Vector Spaces	352
	Problems	373
2.5.3	Conic Sections	375

Problems	387
2.5.4 Polar Coordinates	388
Problems	394
2.5.5 Quadric Surfaces	395
Problems	401
2.5.6 Cylindrical and Spherical Coordinates	403
Problems	406
2.5.7 Limits in \mathbb{R}^n	408
Problems	411
2.5.8 Paths in \mathbb{R}^n	412
Problems	426
3 Calculus III	429
3.1 Vector-Valued Functions of Several Variables	429
Problems	444
3.2 Derivatives of Vector-Valued Functions of Several Variables	447
Problems	468
3.3 Applications of Differentiation	473
Problems	494
3.4 Integration of Functions of Several Variables	496
Problems	530
3.5 Vector Calculus	538
Problems	548
3.6 Generalizations of the Fundamental Theorem of Calculus	550
3.6.1 Green's Theorem	555
3.6.2 Stokes' Theorem	569
3.6.3 Gauss' Theorem	581
Problems	590
Appendix A	597
A.1 Construction of the Real-Number System	597
A.2 The Lebesgue Criterion for Riemann Integrability	606
A.3 Properties of the Determinant	610
A.4 The Inverse Mapping Theorem	623
References	629
Index of Notation	635
Index of Terminology	637

