

## FULL CONTENTS

<b>PREFACE</b>	xvi
<b>LIST OF EDITORS AND CONTRIBUTORS</b>	xvii
<b>1 Geometry and physics: a personal view</b>	1
<i>Nigel Hitchin</i>	
<b>2 Mathematical work of Nigel Hitchin</b>	11
<i>Sir Michael Atiyah</i>	
<b>3 The Einstein–Maxwell equations, extremal Kähler metrics, and Seiberg–Witten theory</b>	17
<i>Claude LeBrun</i>	
<b>4 The Nahm transform for calorons</b>	34
<i>Benoit Charbonneau and Jacques Hurtubise</i>	
4.1 Introduction	34
4.2 The work of Nye and Singer	35
4.2.1 Two types of invariant self-dual gauge fields on $\mathbb{R}^4$	35
4.2.2 The Nahm transform	37
4.2.3 Involutivity of the transforms	39
4.3 Twistor transform for calorons/Kač–Moody monopoles	39
4.3.1 Upstairs: twistor transform for calorons	39
4.3.2 Downstairs: caloron as a Kač–Moody monopole	40
4.4 From Nahm’s equations to spectral data, and back	46
4.4.1 Flows of sheaves	46
4.4.2 Boundary conditions	48
4.5 Closing the circle	55
4.5.1 Starting with a caloron	55
4.5.2 Starting with a solution to Nahm’s equation	58
4.5.3 From Nahm to caloron to Nahm to caloron	64
4.6 Moduli	66
<b>5 Nahm’s equations and free-boundary problems</b>	71
<i>Simon K. Donaldson</i>	
5.1 Introduction	71
5.2 An infinite-dimensional Riemannian manifold	74

5.3	Three equivalent problems	78
5.3.1	$\theta$ equation $\Rightarrow \phi$ equation	79
5.3.2	$\Phi$ equation $\Rightarrow U$ equation	81
5.3.3	$U$ equation $\Rightarrow \theta$ equation	81
5.4	Existence results and discussion	82
5.4.1	Monge–Ampère and the results of Chen	82
5.4.2	Comparison with the free-boundary literature	84
5.4.3	Degenerate case	86
5.5	Relation with Nahm’s equations	86
<b>6</b>	<b>Some aspects of the theory of Higgs pairs</b>	92
	<i>S. Ramanan</i>	
6.1	Moduli of vector bundles	92
6.2	Hecke correspondence	94
6.3	Moduli of Higgs pairs	95
6.4	Higgs pairs and the fundamental group	97
6.5	Non-abelian Hodge theory	98
6.6	Hitchin morphism	99
6.7	Quantization	101
6.8	Hecke transformation and Hitchin discriminant	102
6.9	Hitchin component	103
6.10	Reductive groups and principal bundles	105
6.11	Reductive groups and Higgs pairs	106
6.12	Real forms	108
<b>7</b>	<b>Mirror symmetry, Hitchin’s equations, and Langlands duality</b>	113
	<i>Edward Witten</i>	
7.1	<i>A</i> -model and <i>B</i> -model	113
7.2	Mirror symmetry and Hitchin’s equations	114
7.3	Hitchin fibration	116
7.3.1	A few hints	117
7.4	Ramification	118
7.5	Wild ramification	120
7.6	Four-dimensional gauge theory and stacks	123
7.6.1	Stacks	125
<b>8</b>	<b>Higgs bundles and geometric structures on surfaces</b>	129
	<i>William M. Goldman</i>	
8.1	Introduction	129
8.2	Representations of the fundamental group	130
8.2.1	Closed surface groups	130
8.2.2	Representation variety	130
8.2.3	Symmetries	132
8.2.4	Deformation space	132

8.3	Abelian groups and rank 1 Higgs bundles	133
8.3.1	Symplectic vector spaces	133
8.3.2	Multiplicative characters: $G = \mathbb{C}^*$	133
8.3.3	Jacobi variety of a Riemann surface	134
8.4	Stable vector bundles and Higgs bundles	135
8.5	Hyperbolic geometry: $G = \mathrm{PSL}(2, \mathbb{R})$	137
8.5.1	Geometric structures	137
8.5.2	Relation to the fundamental group	138
8.5.3	Examples of hyperbolic structures	138
8.6	Moduli of hyperbolic structures and representations	142
8.6.1	Deformation spaces of geometric structures	142
8.6.2	Fuchsian components of $\mathrm{Hom}(\pi, G)/G$	143
8.6.3	Characteristic classes and maximal representations	144
8.6.4	Quasi-Fuchsian representations: $G = \mathrm{PSL}(2, \mathbb{C})$	145
8.6.5	Teichmüller space: marked conformal structures	148
8.6.6	Holomorphic vector bundles and uniformization	149
8.7	Rank 2 Higgs bundles	150
8.7.1	Harmonic metrics	150
8.7.2	Higgs pairs and branched hyperbolic structures	151
8.7.3	Uniformization with singularities	152
8.8	Split $\mathbb{R}$ -forms and Hitchin's Teichmüller component	153
8.8.1	Convex $\mathbb{RP}^2$ -structures: $G = \mathrm{SL}(3, \mathbb{R})$	153
8.8.2	Higgs bundles and affine spheres	154
8.8.3	Hyperconvex curves	155
8.9	Hermitian symmetric spaces: maximal representations	156
8.9.1	Unitary groups $U(p, q)$	156
8.9.2	Symplectic groups $Sp(n, \mathbb{R})$	157
8.9.3	Geometric structures associated to Hitchin representations	158
<b>9</b>	<b>Locality of holomorphic bundles, and locality in quantum field theory</b>	164
	<i>Graeme Segal</i>	
9.1	Noncommutative geometry	165
9.2	Algebraic structures up to homotopy	169
9.3	Quantum field theory	171
<b>10</b>	<b>Toeplitz operators and Hitchin's projectively flat connection</b>	177
	<i>Jørgen Ellegaard Andersen</i>	
10.1	Introduction	177
10.1.1	Geometric construction of $Z_k^{(n)}$	179
10.1.2	Asymptotic faithfulness	181
10.1.3	Kashdan's property (T) and the mapping class group	182
10.1.4	Geometric construction of the curve operators	184

10.1.5 The Nielsen–Thurston classification of mapping classes is determined by TQFT	185
10.1.6 General setting	188
10.2 The Hitchin connection	188
10.3 Toeplitz operators and Berezin–Toeplitz deformation quantization	194
10.4 The formal Hitchin connection	198
10.5 Asymptotics flatness of Toeplitz operators	201
10.6 General asymptotic faithfulness	204
<b>11 Towards a non-linear Schwarz’s list</b>	210
<i>Philip Boalch</i>	
11.1 Introduction	210
11.1.1 Naive generalizations	211
11.1.2 Non-linear analogue: the Painlevé VI equation	213
11.2 What is Painlevé VI?	214
11.2.1 Conceptual approach to Painlevé VI	216
11.2.2 Explicit non-linear equations	217
11.2.3 Monodromy of Painlevé VI	218
11.3 Algebraic solutions from finite subgroups of $\mathbf{SL}_2(\mathbb{C})$	219
11.3.1 What exactly is an algebraic solution?	219
11.3.2 (A) : –(C)	219
11.4 Beyond Platonic Painlevé VI solutions	221
11.4.1 Construction	222
11.4.2 Relating connections (A) and (B)	223
11.4.3 New solutions	226
11.5 Pullbacks	228
11.6 Final steps	231
11.6.1 Up to degree 24	231
11.6.2 Quadratic/Landen/folding transformations	232
11.7 Conclusion	233
<b>12 An introduction to bundle gerbes</b>	237
<i>Michael K. Murray</i>	
12.1 Introduction	238
12.2 Background	242
12.3 Bundle gerbes	243
12.3.1 Pullback	245
12.3.2 Dual and product	245
12.3.3 Characteristic class	245
12.3.4 Connective structure	246
12.4 Triviality	247
12.4.1 Holonomy	250
12.4.2 Obstructions to certain kinds of $Y \rightarrow M$	251

12.5 Examples of bundle gerbes	251
12.5.1 Lifting bundle gerbe	252
12.5.2 Projective bundles	253
12.5.3 Bundle gerbes on Lie groups	254
12.6 Applications of bundle gerbes	255
12.6.1 Wess–Zumino–Witten term	255
12.6.2 Faddeev–Mickelsson anomaly	256
12.6.3 String structures	257
12.7 Other matters	257
<b>13 Projective linking and boundaries of positive holomorphic chains in projective manifolds, part I</b>	261
<i>F. Reese Harvey and H. Blaine Lawson, Jr.</i>	
13.1 Introduction	261
13.2 Projective hulls	264
13.3 Projective linking and projective winding numbers	267
13.4 Quasi-plurisubharmonic functions	269
13.5 Boundaries of positive holomorphic chains	270
13.6 Projective Alexander–Wermer theorem for curves	271
13.7 Theorems for general projective manifolds	275
13.8 Relative holomorphic cycles	277
<b>14 Skyrmions and nuclei</b>	281
<i>Nicholas S. Manton</i>	
14.1 Skyrmions	281
14.2 Rational map ansatz	283
14.3 Skyrmions and $\alpha$ -particles	286
14.3.1 $B = 8$	286
14.3.2 $B = 12$	287
14.3.3 $B = 16$	289
14.3.4 $B = 24$	290
14.3.5 $B = 28$	291
14.3.6 $B = 32$	291
14.4 Quantization and calibration	292
14.5 Conclusions	294
Appendix: Double rational map ansatz	295
<b>15 Mirror symmetry of Fourier–Mukai transformation for elliptic Calabi–Yau manifolds</b>	299
<i>Naichung Conan Leung and Shing-Tung Yau</i>	
15.1 Introduction	299
15.2 Mirror symmetry and SYZ transformation	300
15.2.1 Geometry of Calabi–Yau manifolds	300
15.2.2 Mirror symmetry conjectures	300

15.2.3 SYZ transform	301
15.2.4 Mirror of elliptic fibrations	302
15.3 Fourier–Mukai transform and elliptic CY manifolds	304
15.3.1 General Fourier–Mukai transform	304
15.3.2 Elliptic fibrations and their duals	304
15.3.3 FM transform and spectral cover construction	305
15.4 Symplectic FM transform and twin Lagrangian fibrations	306
15.4.1 Symplectic Fourier–Mukai transform	306
15.4.2 Twin Lagrangian fibrations	307
15.4.3 Dual twin Lagrangian fibration and its FM transform	312
15.4.4 Examples of twin Lagrangian fibrations	313
15.5 SYZ transformation of FM transform	315
15.5.1 SYZ transform of dual elliptic fibrations	315
15.5.2 SYZ transform of the universal Poincaré sheaf	317
15.5.3 SYZ transforms commute with FM transforms	320
15.6 Conclusions and discussions	321
<b>16 S-duality in hyperkähler Hodge theory</b>	324
<i>Tamás Hausel</i>	
16.1 Introduction	324
16.2 Hyperkähler quotients	326
16.2.1 Moduli of Yang–Mills instantons on $\mathbb{R}^4$	326
16.2.2 Moduli space of magnetic monopoles on $\mathbb{R}^3$	327
16.2.3 Hitchin moduli space	328
16.3 Hodge theory	328
16.3.1 $L^2$ harmonic forms on complete manifolds	328
16.3.2 Results on $L^2$ harmonic forms	329
16.4 Mixed Hodge theory	331
16.4.1 Mixed Hodge structure of Deligne	331
16.4.2 Arithmetic and topological content of the $E$ -polynomial	332
16.5 Applications of mixed Hodge theory	333
16.5.1 Nakajima quiver varieties	333
16.5.2 Spaces diffeomorphic to the Hitchin moduli space $\mathcal{M}(C, P_{U(n)})$	336
16.5.3 Topological mirror test	337
16.5.4 Mirror symmetry for finite groups of Lie type	338
16.5.5 Conjectural answer	339
<b>17 Non-embedding and non-extension results in special holonomy</b>	346
<i>Robert L. Bryant</i>	
17.1 Introduction	346
17.2 Beginnings	349
17.2.1 Holonomy	349

17.2.2 A non-trivial case	350
17.2.3 Hyper-Kähler viewpoint	351
<b>17.3 Hyper-Kähler four-manifolds</b>	<b>353</b>
17.3.1 An exterior differential systems proof	353
17.3.2 A sharper result	353
<b>17.4 <math>G_2</math>-Manifolds</b>	<b>358</b>
17.4.1 Hypersurfaces	359
17.4.2 Flow interpretation	362
<b>17.5 Spin(7)-manifolds</b>	<b>362</b>
17.5.1 Hypersurfaces	364
17.5.2 Flow interpretation	365
<b>18 Branes on Poisson varieties</b>	<b>368</b>
<i>Marco Gualtieri</i>	
18.1 Introduction	368
18.2 Gerbe trivializations	369
18.3 Generalized connections	370
18.4 Generalized holomorphic bundles and branes	374
18.4.1 Generalized holomorphic bundles	375
18.4.2 Generalized complex branes	377
18.5 Multiple branes and holomorphic Poisson varieties	380
18.6 Relation to generalized Kähler geometry	385
18.7 Construction of generalized Kähler metrics	388
18.8 Relation to non-commutative algebraic geometry	391
<b>19 Consistent orientation of moduli spaces</b>	<b>395</b>
<i>Daniel S. Freed, Michael J. Hopkins, and Constantin Teleman</i>	
19.1 Push-pull construction of TQFT	397
19.1.1 Quantum field theory	397
19.1.2 Topological construction	398
19.1.3 Remarks	400
19.2 Orientation and twisting	401
19.2.1 Ordinary cohomology	401
19.2.2 $K$ -theory	403
19.3 Universal orientations and consistent orientations	405
19.3.1 Overview	405
19.3.2 Closed surfaces	405
19.3.3 Surfaces with boundary	408
19.3.4 The level	411
19.3.5 Pushforward maps	413
19.4 Families of surfaces, twistings, and anomalies	415
<b>INDEX</b>	<b>421</b>