Contents

Introduction xi

1 Essentials of linear algebra 3
1.1 Motivating problems 3
1.2 Systems of linear equations 8
1.2.1 Row reduction using *Maple* 15
1.3 Linear combinations 21
1.3.1 Markov chains: an application of matrix-vector multiplication 26
1.3.2 Matrix products using *Maple* 29
1.4 The span of a set of vectors 33
1.5 Systems of linear equations revisited 39
1.6 Linear independence 49
1.7 Matrix algebra 58
1.7.1 Matrix algebra using *Maple* 62
1.8 The inverse of a matrix 66
1.8.1 Computer graphics 70
1.8.2 Matrix inverses using *Maple* 73
1.9 The determinant of a matrix 78
1.9.1 Determinants using *Maple* 82
1.10 The eigenvalue problem 84
1.10.1 Markov chains, eigenvectors, and Google 93
1.10.2 Using *Maple* to find eigenvalues and eigenvectors 94

1.11 Generalized vectors 99
1.12 Bases and dimension in vector spaces 108
1.13 For further study 115
1.13.1 Computer graphics: geometry and linear algebra at work 115
1.13.2 Bézier curves 119
1.13.3 Discrete dynamical systems 123

2 First-order differential equations 127
2.1 Motivating problems 127
2.2 Definitions, notation, and terminology 129
2.2.1 Plotting slope fields using *Maple* 135
2.3 Linear first-order differential equations 139
2.4 Applications of linear first-order differential equations 147
2.4.1 Mixing problems 147
2.4.2 Exponential growth and decay 148
2.4.3 Newton's law of Cooling 150
2.5 Nonlinear first-order differential equations 154
2.5.1 Separable equations 154
2.5.2 Exact equations 157
2.6 Euler's method 162
2.6.1 Implementing Euler's method in Excel 167
2.7 Applications of nonlinear first-order differential equations 172
2.7.1 The logistic equation 172
2.7.2 Torricelli's law 176
2.8 For further study 181
2.8.1 Converting certain second-order des to first-order DEs 181
2.8.2 How raindrops fall 182
2.8.3 Riccati's equation 183
2.8.4 Bernoulli's equation 184

3 Linear systems of differential equations 187
3.1 Motivating problems 187
3.2 The eigenvalue problem revisited 191
3.3 Homogeneous linear first-order systems 202
3.4 Systems with all real linearly independent eigenvectors 211
3.4.1 Plotting direction fields for systems using *Maple* 219
3.5 When a matrix lacks two real linearly independent eigenvectors 223
3.6 Nonhomogeneous systems: undetermined coefficients 236
3.7 Nonhomogeneous systems: variation of parameters 245
3.7.1 Applying variation of parameters using *Maple* 250

3.8 Applications of linear systems 253
3.8.1 Mixing problems 253
3.8.2 Spring-mass systems 255
3.8.3 RLC circuits 258
3.9 For further study 268
3.9.1 Diagonalizable matrices and coupled systems 268
3.9.2 Matrix exponential 270

4 Higher order differential equations 273
4.1 Motivating equations 273
4.2 Homogeneous equations: distinct real roots 274
4.3 Homogeneous equations: repeated and complex roots 281
4.3.1 Repeated roots 281
4.3.2 Complex roots 283
4.4 Nonhomogeneous equations 288
4.4.1 Undetermined coefficients 289
4.4.2 Variation of parameters 295
4.5 Forced motion: beats and resonance 300
4.6 Higher order linear differential equations 309
4.6.1 Solving characteristic equations using *Maple* 316
4.7 For further study 319
4.7.1 Damped motion 319
4.7.2 Forced oscillations with damping 321
4.7.3 The Cauchy–Euler equation 323
4.7.4 Companion systems and companion matrices 325

5 Laplace transforms 329
5.1 Motivating problems 329
5.2 Laplace transforms: getting started 331
5.3 General properties of the Laplace transform 337
5.4 Piecewise continuous functions 347
5.4.1 The Heaviside function 347
5.4.2 The Dirac delta function 353
5.4.3 The Heaviside and Dirac functions in *Maple* 357
5.5 Solving IVPs with the Laplace transform 359
5.6 More on the inverse Laplace transform 371
5.6.1 Laplace transforms and inverse transforms using *Maple* 375
5.7 For further study 378
5.7.1 Laplace transforms of infinite series 378
5.7.2 Laplace transforms of periodic forcing functions 380
5.7.3 Laplace transforms of systems 384

6 Nonlinear systems of differential equations 387
6.1 Motivating problems 387

6.2 Graphical behavior of solutions for 2 × 2 nonlinear systems 391
6.2.1 Plotting direction fields of nonlinear systems using *Maple* 397
6.3 Linear approximations of nonlinear systems 400
6.4 Euler's method for nonlinear systems 409
6.4.1 Implementing Euler's method for systems in Excel 413
6.5 For further study 417
6.5.1 The damped pendulum 417
6.5.2 Competitive species 418

7 Numerical methods for differential equations 421
7.1 Motivating problems 421
7.2 Beyond Euler's method 423
7.2.1 Heun's method 424
7.2.2 Modified Euler's method 427
7.3 Higher order methods 430
7.3.1 Taylor methods 431
7.3.2 Runge–Kutta methods 434
7.4 Methods for systems and higher order equations 439
7.4.1 Euler's method for systems 440
7.4.2 Heun's method for systems 442
7.4.3 Runge–Kutta method for systems 443
7.4.4 Methods for higher order IVPs 445
7.5 For further study 449
7.5.1 Predator–Prey equations 449
7.5.2 Competitive species 450
7.5.3 The damped pendulum 450

8 Series solutions for differential equations 453
8.1 Motivating problems 453
8.2 A review of Taylor and power series 455
8.3 Power series solutions of linear equations 463
8.4 Legendre's equation 471
8.5 Three important examples 477
8.5.1 The Hermite equation 477
8.5.2 The Laguerre equation 480
8.5.3 The Bessel equation 482
8.6 The method of Frobenius 485
8.7 For further study 491
8.7.1 Taylor series for first-order differential equations 491
8.7.2 The Gamma function 491

Appendix A	Review of integration techniques	493
Appendix B	Complex numbers	503
Appendix C	Roots of polynomials	509
Appendix D	Linear transformations	513
Appendix E	Solutions to selected exercises	523
Index		549