## Contents

1 What can't be ignored ..... 1
1.1 The MATLAB and Octave environments ..... 1
1.2 Real numbers ..... 3
1.2.1 How we represent them ..... 3
1.2.2 How we operate with floating-point numbers ..... 6
1.3 Complex numbers ..... 8
1.4 Matrices ..... 10
1.4.1 Vectors ..... 14
1.5 Real functions ..... 16
1.5.1 The zeros ..... 18
1.5.2 Polynomials ..... 20
1.5.3 Integration and differentiation ..... 22
1.6 To err is not only human ..... 25
1.6.1 Talking about costs ..... 29
1.7 The MATLAB language ..... 30
1.7.1 MATLAB statements ..... 32
1.7.2 Programming in MATLAB ..... 34
1.7.3 Examples of differences between MATLAB and Octave languages ..... 37
1.8 What we haven't told you ..... 38
1.9 Exercises ..... 38
2 Nonlinear equations ..... 41
2.1 Some representative problems ..... 41
2.2 The bisection method ..... 43
2.3 The Newton method ..... 47
2.3.1 How to terminate Newton's iterations ..... 49
2.3.2 The Newton method for systems of nonlinear equations ..... 51
2.4 Fixed point iterations ..... 54
2.4.1 How to terminate fixed point iterations ..... 60
2.5 Acceleration using Aitken's method ..... 60
2.6 Algebraic polynomials ..... 65
2.6.1 Hörner's algorithm ..... 66
2.6.2 The Newton-Hörner method ..... 68
2.7 What we haven't told you ..... 70
2.8 Exercises ..... 72
3 Approximation of functions and data ..... 75
3.1 Some representative problems ..... 75
3.2 Approximation by Taylor's polynomials ..... 77
3.3 Interpolation ..... 78
3.3.1 Lagrangian polynomial interpolation ..... 79
3.3.2 Stability of polynomial interpolation ..... 84
3.3.3 Interpolation at Chebyshev nodes ..... 86
3.3.4 Trigonometric interpolation and FFT ..... 88
3.4 Piecewise linear interpolation ..... 93
3.5 Approximation by spline functions ..... 94
3.6 The least-squares method ..... 99
3.7 What we haven't told you ..... 103
3.8 Exercises ..... 105
4 Numerical differentiation and integration ..... 107
4.1 Some representative problems ..... 107
4.2 Approximation of function derivatives ..... 109
4.3 Numerical integration ..... 111
4.3.1 Midpoint formula ..... 112
4.3.2 Trapezoidal formula ..... 114
4.3.3 Simpson formula ..... 115
4.4 Interpolatory quadratures ..... 117
4.5 Simpson adaptive formula ..... 121
4.6 What we haven't told you ..... 125
4.7 Exercises ..... 126
5 Linear systems ..... 129
5.1 Some representative problems ..... 129
5.2 Linear system and complexity ..... 134
5.3 The LU factorization method ..... 135
5.4 The pivoting technique ..... 144
5.5 How accurate is the solution of a linear system? ..... 147
5.6 How to solve a tridiagonal system ..... 150
5.7 Overdetermined systems ..... 152
5.8 What is hidden behind the MATLAB command $\backslash$ ..... 154
5.9 Iterative methods ..... 157
5.9.1 How to construct an iterative method ..... 158
5.10 Richardson and gradient methods ..... 162
5.11 The conjugate gradient method ..... 166
5.12 When should an iterative method be stopped? ..... 169
5.13 To wrap-up: direct or iterative? ..... 171
5.14 What we haven't told you ..... 177
5.15 Exercises ..... 177
6 Eigenvalues and eigenvectors ..... 181
6.1 Some representative problems ..... 182
6.2 The power method ..... 184
6.2.1 Convergence analysis ..... 187
6.3 Generalization of the power method ..... 188
6.4 How to compute the shift ..... 190
6.5 Computation of all the eigenvalues ..... 193
6.6 What we haven't told you ..... 197
6.7 Exercises ..... 197
7 Ordinary differential equations ..... 201
7.1 Some representative problems ..... 201
7.2 The Cauchy problem ..... 204
7.3 Euler methods ..... 205
7.3.1 Convergence analysis ..... 208
7.4 The Crank-Nicolson method ..... 212
7.5 Zero-stability ..... 214
7.6 Stability on unbounded intervals ..... 216
7.6.1 The region of absolute stability ..... 219
7.6.2 Absolute stability controls perturbations ..... 220
7.7 High order methods ..... 228
7.8 The predictor-corrector methods ..... 234
7.9 Systems of differential equations ..... 236
7.10 Some examples ..... 242
7.10.1 The spherical pendulum ..... 242
7.10.2 The three-body problem ..... 246
7.10.3 Some stiff problems ..... 248
7.11 What we haven't told you ..... 252
7.12 Exercises ..... 252
8 Numerical approximation of boundary-value problems ..... 255
8.1 Some representative problems ..... 256
8.2 Approximation of boundary-value problems ..... 258
8.2.1 Finite difference approximation of the one-dimensional Poisson problem ..... 259
8.2.2 Finite difference approximation of a convection-dominated problem ..... 262
8.2.3 Finite element approximation of the one-dimensional Poisson problem ..... 263
8.2.4 Finite difference approximation of the two-dimensional Poisson problem ..... 267
8.2.5 Consistency and convergence of finite difference discretization of the Poisson problem ..... 272
8.2.6 Finite difference approximation of the one-dimensional heat equation ..... 274
8.2.7 Finite element approximation of the one-dimensional heat equation ..... 278
8.3 Hyperbolic equations: a scalar pure advection problem ..... 281
8.3.1 Finite difference discretization of the scalar transport equation ..... 283
8.3.2 Finite difference analysis for the scalar transport equation ..... 285
8.3.3 Finite element space discretization of the scalar advection equation ..... 292
8.4 The wave equation ..... 293
8.4.1 Finite difference approximation of the wave equation ..... 295
8.5 What we haven't told you ..... 299
8.6 Exercises ..... 300
9 Solutions of the exercises ..... 303
9.1 Chapter 1 ..... 303
9.2 Chapter 2 ..... 306
9.3 Chapter 3 ..... 312
9.4 Chapter 4 ..... 315
9.5 Chapter 5 ..... 320
9.6 Chapter 6 ..... 327
9.7 Chapter 7 ..... 330
9.8 Chapter 8 ..... 339
References ..... 347
Index ..... 353

