

Contents

<i>Preface</i>	<i>page ix</i>
1 Preliminaries	1
1.1 Notation	1
1.2 Monoids	2
1.3 Words	4
1.4 Automata	11
1.5 Transducers	19
1.6 Semirings and matrices	20
1.7 Formal series	23
1.8 Power series	26
1.9 Nonnegative matrices	28
1.10 Weighted automata	32
1.11 Probability distributions	39
1.12 Ideals in a monoid	41
1.13 Permutation groups	48
1.14 Notes	52
2 Codes	55
2.1 Definitions	55
2.2 Codes and free submonoids	60
2.3 A test for codes	67
2.4 Codes and Bernoulli distributions	71
2.5 Complete sets	75
2.6 Composition	86
2.7 Prefix graph of a code	93
2.8 Exercises	100
2.9 Notes	103
3 Prefix codes	107
3.1 Prefix codes	107
3.2 Automata	113

3.3	Maximal prefix codes	120
3.4	Operations on prefix codes	123
3.5	Semaphore codes	131
3.6	Synchronized codes	137
3.7	Recurrent events	145
3.8	Length distributions	152
3.9	Optimal prefix codes	158
3.10	Exercises	170
3.11	Notes	174
4	Automata	177
4.1	Unambiguous automata	177
4.2	Flower automaton	182
4.3	Decoders	191
4.4	Exercises	197
4.5	Notes	198
5	Deciphering delay	199
5.1	Deciphering delay	199
5.2	Maximal codes	203
5.3	Weakly prefix codes	213
5.4	Exercises	219
5.5	Notes	223
6	Bifix codes	225
6.1	Basic properties	226
6.2	Maximal bifix codes	231
6.3	Degree	237
6.4	Kernel	248
6.5	Finite maximal bifix codes	254
6.6	Completion	263
6.7	Exercises	269
6.8	Notes	273
7	Circular codes	275
7.1	Circular codes	275
7.2	Limited codes	281
7.3	Length distributions	286
7.4	Exercises	297
7.5	Notes	298
8	Factorizations of free monoids	301
8.1	Factorizations	301
8.2	Finite factorizations	313
8.3	Exercises	323
8.4	Notes	325

9 Unambiguous monoids of relations	327
9.1 Unambiguous monoids of relations	328
9.2 The Schützenberger representations	336
9.3 Rank and minimal ideal	343
9.4 Very thin codes	349
9.5 Group and degree of a code	358
9.6 Interpretations	360
9.7 Exercises	363
9.8 Notes	370
10 Synchronization	373
10.1 Synchronizing pairs	373
10.2 Uniformly synchronized codes	377
10.3 Locally parsable codes and local automata	382
10.4 Road coloring	388
10.5 Exercises	394
10.6 Notes	395
11 Groups of codes	397
11.1 Groups and composition	397
11.2 Synchronization of semaphore codes	404
11.3 Group codes	410
11.4 Automata of bifix codes	412
11.5 Depth	416
11.6 Groups of finite bifix codes	418
11.7 Examples	425
11.8 Exercises	430
11.9 Notes	433
12 Factorizations of cyclic groups	435
12.1 Factorizations of cyclic groups	435
12.2 Bayonets	439
12.3 Hooks	445
12.4 Exercises	447
12.5 Notes	449
13 Densities	451
13.1 Probability	451
13.2 Densities	460
13.3 Entropy	467
13.4 Probabilities over a monoid	470
13.5 Strict contexts	480
13.6 Exercises	489
13.7 Notes	490

14 Polynomials of finite codes	493
14.1 Positive factorizations	493
14.2 The factorization theorem	497
14.3 Noncommutative polynomials	499
14.4 Proof of the factorization theorem	505
14.5 Applications	509
14.6 Commutative equivalence	512
14.7 Complete reducibility	520
14.8 Exercises	530
14.9 Notes	534
<i>Solutions of exercises</i>	535
<i>Appendix: Research problems</i>	591
<i>References</i>	594
<i>Index of notation</i>	609
<i>Index</i>	611