Preface

New? In what sense? Surely I am not the only person who, after extensively justifying why certain mathematical structures naturally arise in physics, gets questions like: “this is all nice maths but what’s the physics?” Meanwhile I figured out what this truly means: “I don’t see any differential equations!” Okay, this is indeed a bit overstated. Nowadays any mathematical argument involving groups, when these are moreover referred to as “symmetry groups”, stands a serious chance of being eligible for carrying the label “physics”. But it hasn’t always been like this. John Slater (cf. the Slater determinant in quantum chemistry) referred to the use of group theory in quantum physics by Weyl, Wigner et al. as der Gruppenpest, what translates as the “plague of groups”. Even in 1975 he wrote [14]: “As soon as [my] paper became known, it was obvious that a great many other physicists were as ‘disgusted’ as I had been with the group-theoretical approach to the problem. As I heard later, there were remarks made such as ‘Slater has slain the Gruppenpest’. I believe that no other piece of work I have done was so universally popular.” Donkeys usually don’t make the same mistake twice, . . .

. . . and, surely I am not the only person who, after extensively justifying why certain mathematical structures naturally arise in physics, gets questions like: “this is just the same thing in a different language!” Well, so was Copernicus’ description of the planets as compared to Ptolemy’s. Looking back at the facts, Ptolemy’s description turned out to be more accurate, accounting even for relativistic effects. So was abandoning the view that the Earth was the centre of the universe and that planets move around it on hierarchies of epicycles a step backward? Of course not. Taking the superheavy object that the sun is to be a fixed point of reference unveiled the gravitational force as well as a critical glimpse of Newton’s laws of motion, in terms of Galilei’s visions and Kepler’s work. Similarly, programming languages are not just a different way of writing down 0s and 1s, but also capture the flows of information within a computational process. Surely we wouldn’t want the whole of mathematics to be written down entirely in terms of 0s and 1s; imagine researching physics in terms of nothing but 0s and 1s! All this just to say that language means structure and additional structure means additional content: using group theory is not just about using a different language but about identifying symmetry as a key ingredient of physics. The same goes for the structures that are discussed in this book: they all identify a key ingredient in physics that deserves our attention. They moreover identify this ingredient as present in a wide range of theories, including theories of information and computation.

The contributing research landscape. Once a subset of mathematics is accepted by the general physics community as relevant, many physicists seem to stop making a distinction between that piece of mathematics and the natural phenomenon this piece of mathematics aims to describe. For this reason, there is a high entrance fee for a mathematical structure to be awarded this privilege. But this also means that progress in physics does go hand-in-hand with the use of new mathematical structures. This book contains a number of such structures which recently have been finding their way into quantum information, foundations of general relativity, quantum foundations, and quantum gravity foundations. A surprising feature of many of these is that these structures are already heavily used in "Euro-style" computer science, and some were even crafted for this particular purpose. In general relativity "Scott domains" enable to reconstruct spacetime topology from the causal structure without making any reference to smoothness [10]. Dana Scott (= male) initially introduced these domains in the late 60s to provide semantics for the λ-calculus [13], which plays a key role both in the foundations of mathematics and in programming language foundations [19]. In quantum information monoidal categories [21] are becoming more prominent, for example, for the description of particular computational models such as topological quantum computing (see [11] for a survey), and measurement-based quantum computing (see for example [5, 6, 8]), in which the interaction between classical and quantum data is of key importance. Earlier it was already suggested that topological quantum field theories [2], which are functors between certain kinds of monoidal categories, could be relevant for a theory of quantum gravity [3, 7]. Again, these monoidal categories are of key importance in computer science, for example, they provide semantics for linear logic [20], a logic which is important in concurrency theory [18], the theoretical underpinning of mobile phone networks, internet protocols, cash machines etc.

At the n-category café John Baez suggested that a less opportunistic title for this volume would have been: "Structures you would already know about, had you been paying proper attention". While as title poetry this isn't great, he is of course right, and for more than one reason. John himself pointed to the fact that, for example, "Category theory has been important in algebraic topology ever since its interception in 1945. It's just taken a while for these structures to become part of the toolkit of the average mathematical physicist." He and Mike Stay have more examples on page 125 of their chapter entitled "Physics, topology, logic and computation: A Rosetta Stone" [4]. The other reason is the one I mentioned above: these structures are already heavily used in theoretical computer science, where the play the role of "logic of interaction" [1], "discrete (relativistic!) spacetime" [9, 12], among many other roles.

A personal appreciation. I started my research career in the late eighties in quantum foundations. If that didn't already guaranty academic suicide, I moreover studied hidden variable theories. After my PhD, in an attempt to save my career, I moved to the dying area of quantum logic within the retiring Geneva group led by Piron. Having become aware of my mistake I moved into pure mathematics, to category theory, an area hated by most non-category-theoretic-mathematicians, within the retiring category theory group at McGill University. The great surprise is that after all of this I am still standing, while many other scholars, far more brighter than I am, lost the battle. The worst carnage in terms of academic careers surely must have taken place in high energy physics [15, 16]. In quantum foundations the academic death-toll is less, but this mostly has to do with the the style quantum theory is taught in most places: "Don't think, just do!", resulting in not many people ending up in quantum foundations. The reason that I ended up surviving must be that although each of $|\text{quantum foundations}\rangle$, $|\text{quantum logic}\rangle$, $|\text{category theory}\rangle$ causes academic disaster, $|\text{quantum categorical logic foundations}\rangle$ proved to be some kind of a hit in European computer science circles where, surprisingly, "foundations" means "cool". In those circles structural research is indeed highly appreciated, the reason being that one simply can't do without. Meanwhile, the membership of our multidisciplinary group here at Oxford University Computing Laboratory [22] has grown to 30, which besides Samson Abramsky and myself now also includes Andreas Döring, and a zoo of DPhil (= Oxford PhD) students with backgrounds in theoretical physics, computer science, pure mathematics, philosophy, engineering, and even linguistics.

How did this all came about? In 2005 I organized an event called *Cats, Kets and Cloisters* (CKC) at Oxford University Computing Laboratory [23]. The event aimed to set the stage for an encounter of researchers studying mathematical structures in computer science, quantum foundations, pure mathematicians including specialists in logic, category theory and knot theory, and quantum informaticians. It in particular included twelve tutorial lectures by leading experts. The success of the conference what witnessed by the fact that since there was no budget to invite speakers, these twelve leading experts all covered there own expenses. Moreover, a chain of similar events [24–26] emerged after CKC, the most recent one being Categories, Quanta and Concepts (CQC) at the Perimeter Institute [27].

But a low in all this was the following. When asked by several PhD students were they could read about "this kind of stuff", there simply wasn't a satisfactory answer. This is were this volume kicks in: it collates a series of tutorials that do the job.

Contributions to this volume. We start with an ABC on monoidal category theory, by Abramsky and Tzevelekos, Baez and Stay, and Coecke and Paquette. These bulky contributions nicely complement each other, the first one being the lecture notes of the category theory course here at Oxford University Computing Laboratory, the second one exemplifying how the same structures arise in very different areas, and the third one establishing that monoidal categories have always been "out there" in physics. The "linear" feature of these categories is then further emphasized, in graphical realm by Selinger, and in computational realm by Haghverdi and Scott. In particular, Selinger's chapter is the first rock-solid comprehensive account on the topic of graphical calculi for monoidal categories, in which he fixes several caveats of the existing literature. Then follows a Blute-Panagaden double which applies the theory to formal distributions and formal Feynman diagrams. After that we have a living Legend, Jim Lambek, who exposes connections between particle physics and mathematical linguistics, an area which he pioneered in the 1950s. Next up is domain theory, starting with a tutorial overview by Martin, followed by a detailed account of the domain-theoretic structure on classical and quantum states by Coecke and Martin. This is then followed by a range of structures dealing with spacetime: first Martin and Panangaden's application of domain theory to general relativity, then Hiley's use of Clifford algebras, and finally Döring and Isham's use of topos theory in an 180 page long mega contribution. We end with applications of monoidal categories in quantum computational models, firstly a general account by Hines, which is followed by Panangaden and Paquette's survey of topological quantum computing.

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