2.6 Mode Coupling for Dispersion Compensation

We will first introduce a standard description of mode coupling and the so-called differential group delay profile (DGD), a powerful tool for a geometrical understanding of PMD. Then possible implementations of polarization mode and chromatic dispersion compensators will be discussed.

2.6.1 Mode Coupling and Differential Group Delay Profiles

In order to be able to compensate polarization mode dispersion (PMD) a suitable equalizer structure must be found. We will derive the necessary properties of polarization transformers and differential group delay (DGD) sections [10]. Transmission fiber span and equalizer are assumed to consist of lossless optical retarders, and isotropic propagation delays are neglected.

The most general retarder is an endless elliptical retarder (ER) as described by (2.239), (2.240). One of its eigenmodes, the fast one for a positive retardation δ, has the normalized Stokes vector

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \cos 2\vartheta \cos 2\varepsilon \\ \sin 2\vartheta \cos 2\varepsilon \\ \sin 2\varepsilon \end{bmatrix} \quad (V_1^2 + V_2^2 + V_3^2 = 1), \tag{2.433}$$

with azimuth angle ϑ and ellipticity angle ε. Relative strengths of 0°, 45° and circular birefringence are given by V_1, V_2 and V_3, respectively. These as well as the retardation $\delta = -\infty...\infty$ can vary endlessly. Angles $2\vartheta = -\infty...\infty$ and $2\varepsilon = -\pi/2...\pi/2$ allow for endless variations of the eigenmodes.

Both Jones and Müller matrices of optical components can be multiplied in reversed order of light propagation to describe their concatenation. This means the following expressions (2.434)–(2.443) are valid for Jones matrices (2.239) and 3×3 rotation matrices (2.240). We use the nomenclature

$$\begin{aligned}
&\text{(endless) phase shifter } (2\vartheta = 0,\ 2\varepsilon = 0): && \mathrm{PS}(\delta = -\infty...\infty), \\
&\text{(finite) mode converter } (2\vartheta = \pi/2,\ 2\varepsilon = 0): && \mathrm{MC}(\delta = 0...\pi), \\
&\text{(endless) Soleil-Babinet compensator } (2\varepsilon = 0): && \mathrm{SBC}(\delta = 0...\pi, 2\vartheta = -\infty...\infty), \\
&\text{(endless) Soleil-Babinet analog } (2\vartheta = \pi/2): && \mathrm{SBA}(\delta = 0...\pi, 2\varepsilon = -\infty...\infty),
\end{aligned} \tag{2.434}$$

to define some special cases of an (endless) elliptical retarder ER. We call the last example a Soleil-Babinet analog because it is related to the familiar Soleil-Babinet compensator, a rotatable waveplate of adjustable retardation, by cyclical shifts of rows and columns of the rotation matrix. An SBA is described by either of

$$\mathrm{SBA}(\delta,\psi)=\begin{bmatrix}\cos\delta/2 & je^{j\psi}\sin\delta/2\\ je^{-j\psi}\sin\delta/2 & \cos\delta/2\end{bmatrix}\qquad \text{Jones matrix,}$$

$$\mathrm{SBA}(\delta,\psi)=\begin{bmatrix}\cos\delta & -\sin\psi\sin\delta & \cos\psi\sin\delta\\ \sin\psi\sin\delta & \cos^2\psi+\sin^2\psi\cos\delta & \cos\psi\sin\psi(1-\cos\delta)\\ -\cos\psi\sin\delta & \cos\psi\sin\psi(1-\cos\delta) & \sin^2\psi+\cos^2\psi\cos\delta\end{bmatrix}$$
$$\text{rotation matrix.} \qquad (2.435)$$

A rotating SBC ($2\vartheta=\Omega t$) converts circular input polarization partly or fully into its orthogonal including a frequency shift by Ω. A "rotating" SBA ($2\varepsilon=\Omega t$) does the same thing to an *x*-polarized input signal. An SBA can endlessly transform *x* polarization at its input into any output polarization or vice versa. An SBA can be replaced by a phase shifter, a mode converter, and another phase shifter of opposite retardation:

$$\mathrm{SBA}(\varphi,2\varepsilon)=\mathrm{PS}(2\varepsilon)\mathrm{MC}(\varphi)\mathrm{PS}(-2\varepsilon) \qquad (2.436)$$

An ER can be replaced by (or expressed as) a sequence of a quarterwave plate, a halfwave plate, and another quarterwave plate, each of them endlessly rotatable [20], or by an endlessly rotatable halfwave plate and an SBC. Alternatively, an ER can be replaced by a phase shifter, a mode converter, and another phase shifter [21]. It may be shown that the mode converter needs only a finite retardation in the range $0...\pi$. This means the ER can also be substituted by an SBA, preceded or followed by one PS,

$$\begin{aligned}\mathrm{ER}&=\mathrm{PS}(\varphi_3)\mathrm{MC}(\varphi_2)\mathrm{PS}(\varphi_1)\\ &=\mathrm{PS}(\varphi_3+\varphi_1)\mathrm{SBA}(\varphi_2,-\varphi_1)=\mathrm{SBA}(\varphi_2,\varphi_3)\mathrm{PS}(\varphi_1+\varphi_3)\end{aligned}. \qquad (2.437)$$

Comparison of the last two expressions shows that a phase shifter may be "pushed" from left to right through an SBA if the SBA orientation angle is increased by the transferred phase shift.

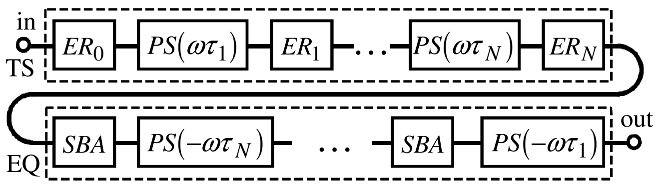


Fig. 2.28 Transmission fiber span (TS) with elliptical retarders (ER) and DGD sections. Suitable equalizer (EQ) with Soleil-Babinet analogs (SBA; see text) and DGD sections, each of which may be accompanied by arbitrary differential phase shifts. © 1999 IEEE.

At an angular optical frequency ω we represent a fiber section having a differential group delay τ by a phase shifter $\mathrm{PS}(\omega\tau)$. Its eigenmodes and principal states-of-polarization (PSP) coincide. The whole transmission span TS may consist of an infinite sequence of alternating ERs and DGD sections. In practice, we limit ourselves to N DGD sections and $N+1$ ERs which are passed by the light signal in ascending order of index i. This is shown in Fig. 2.28. The equalizer (EQ) will be explained later. The matrix describing the transmission fiber span is

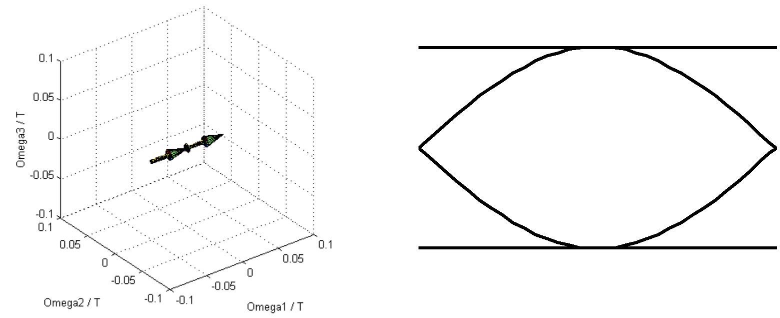
$$\begin{aligned}\mathrm{TS} &= \prod_{i=N}^{1} \left(\mathrm{ER}_i \mathrm{PS}(\omega\tau_i)\right)\mathrm{ER}_0 \\ &= \prod_{i=N}^{1} \left(\mathrm{SBA}(\varphi_{2i},\varphi_{3i})\mathrm{PS}(\varphi_{1i}+\varphi_{3i}+\omega\tau_i)\right)\mathrm{ER}_0\end{aligned} \quad . \tag{2.438}$$

The product symbols with a stop index 1 that is lower than the start index N have to be written from left to right in descending order of index i. Adjacent phase shifts or shifters may be interchanged. All frequency-independent phase shifts in the second expression can therefore be transferred to the right through preceding SBAs according to (2.437),

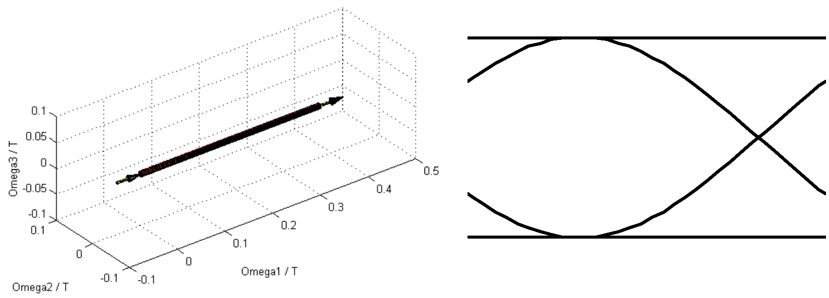
$$\begin{aligned}\mathrm{TS} = &\prod_{i=N}^{1} \left(\mathrm{SBA}\left(\varphi_{2,i},\varphi_{3i}+\sum_{k=i+1}^{N}(\varphi_{1k}+\varphi_{3k})\right)\mathrm{PS}(\omega\tau_i)\right) \\ &\cdot \mathrm{PS}\left(\sum_{i=1}^{N}(\varphi_{1i}+\varphi_{3i})\right)\mathrm{ER}_0\end{aligned} \quad . \tag{2.439}$$

The retarders in the second line can be replaced by another ER.

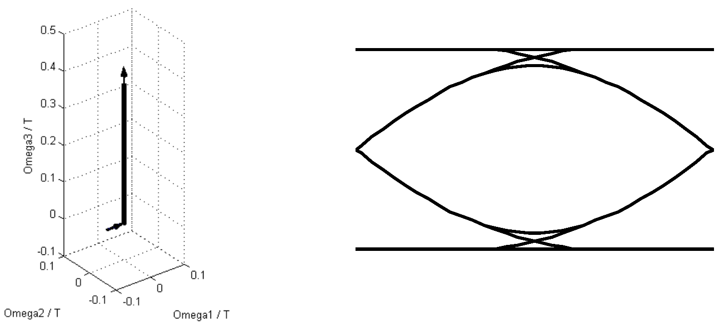
Quite generally the summands $\tilde{\mathbf{\Omega}}_i$ of the overall PMD vector (2.313) of a system can be plotted individually in what may be called the *differential group delay profile*. The input polarization of the first retarder can arbitrarily be set to be horizontal if we multiply its rotation matrix from the right side by a rotation matrix which will transform horizontal into the true retarder input polarization. It is useful to indicate this standardized input polarization by an arrow in $[1 \quad 0 \quad 0]^T$ direction which ends at the origin. From there on the tail of $\tilde{\mathbf{\Omega}}_i$ joins the head of $\tilde{\mathbf{\Omega}}_{i-1}$ and so on. It is always allowed to add a DGD section with zero DGD and a locally horizontal fast PSP at the end of the retarder cascade. Since its PMD vector would have zero length an arrow is plotted instead in the direction of the input-referred PMD vector. This is useful if the retarder cascade is terminated by a frequency-independent retarder. If $\mathbf{G}$ is the total rotation matrix an input polarization $\mathbf{G}^T[1 \quad 0 \quad 0]^T$ would be needed to generate a $[1 \quad 0 \quad 0]^T$ output polarization. So the direction $\mathbf{G}^T[1 \quad 0 \quad 0]^T$ of that output arrow indicates the input polarization which would be necessary to locally hit the fast PSP of that fictitiuos DGD section with zero DGD. The DGD sections have constant DGD τ, so the phase shift between the two PSPs is proportional to $\omega\tau$. Each of the DGD profile rods defined by $\tilde{\mathbf{\Omega}}_i$ therefore twists along its axis if the optical frequency ω is varied. Fig. 2.29 (see legend) shows a number of DGD profiles and associated simulated received eye diagrams. The distortions of Fig. 2.29 e) can be detected by an electrical slope steepness detector [22, 23].



a) 1 (unity matrix)

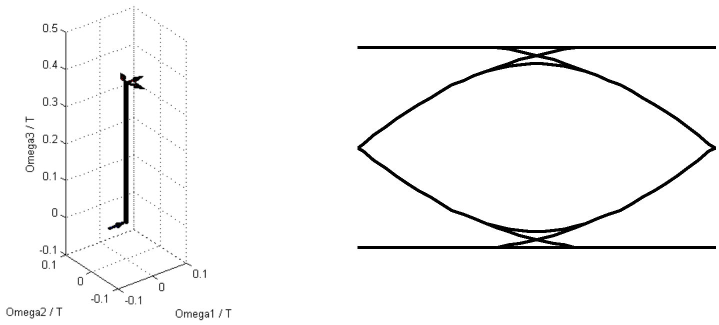


b) $\mathrm{PS}(\omega 3T/8)$

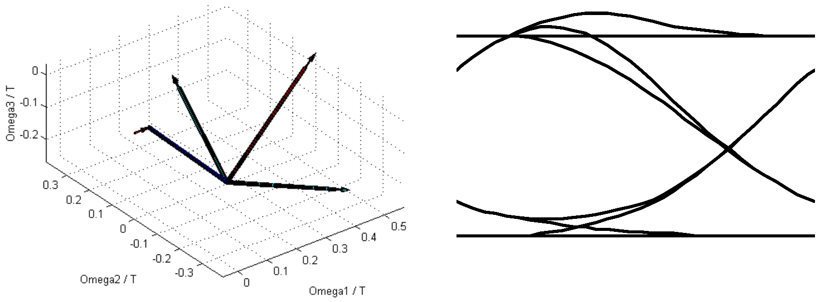


c) $\mathrm{PS}(\omega 3T/8)\mathrm{MC}(\pi/2)$

Fig. 2.29 DGD profiles (left), corresponding simulated NRZ eye diagrams in the receiver (right), and matrix of the investigated retarder chain (below DGD profile) using abbreviations (2.434). Each DGD profile is plotted 3 times: at the center angular frequency ω where we have assumed here that the differential phase shift of each DGD section is an integer of 2π, and with frequency offsets of $\pm 1/(2T)$ where T is the bit duration. The plots at the three different frequencies become distinguishable only from subdiagram d) on.



d) $\mathrm{MC}(-\pi/2)\mathrm{PS}(\omega 3T/8)\mathrm{MC}(\pi/2)$



e) $\mathrm{PS}(\omega 3T/8)\mathrm{MC}(\pi/2)\mathrm{PS}(\omega 3T/8)\mathrm{MC}(-\pi/4)$

Fig. 2.29 (*continued*)

2.6.2 Polarization Mode Dispersion Compensation

A perfect PMD equalizer (EQ or EQ′) has to mirror the PMD profile of the transmission span [10]. Each DGD section of the transmission span will have an oppositely directed, direct neighbor of the equalizer. The principle can be understood from Figs. 2.28 and 2.30.

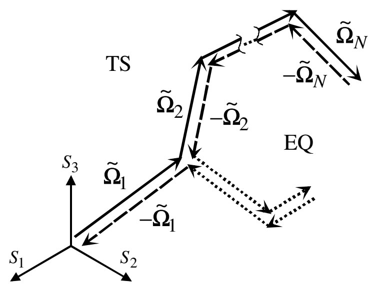


Fig. 2.30 PMD profile of transmission span (solid) and perfect equalizer (dashed, dotted) in the 3-dimensional normalized Stokes or PMD vector space. © 1999 IEEE.