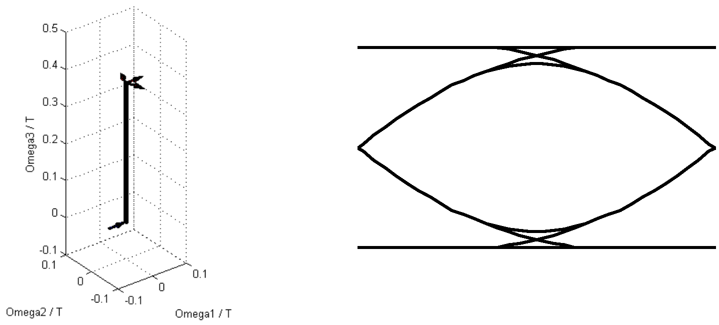
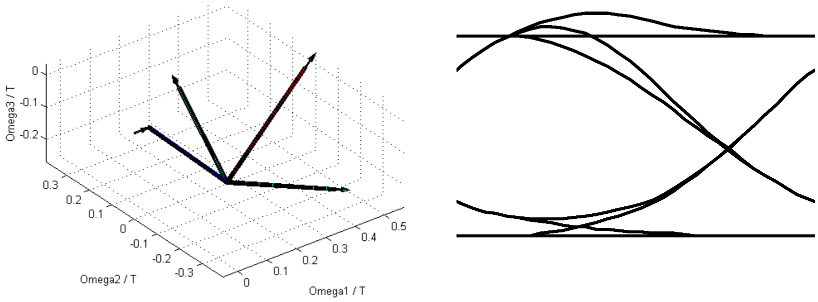


d) $\mathrm{MC}(-\pi/2)\mathrm{PS}(\omega 3T/8)\mathrm{MC}(\pi/2)$



e) $\mathrm{PS}(\omega 3T/8)\mathrm{MC}(\pi/2)\mathrm{PS}(\omega 3T/8)\mathrm{MC}(-\pi/4)$

Fig. 2.29 (*continued*)

2.6.2 Polarization Mode Dispersion Compensation

A perfect PMD equalizer (EQ or EQ′) has to mirror the PMD profile of the transmission span [10]. Each DGD section of the transmission span will have an oppositely directed, direct neighbor of the equalizer. The principle can be understood from Figs. 2.28 and 2.30.

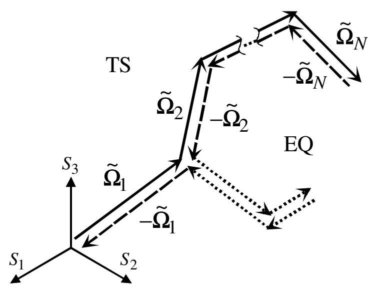


Fig. 2.30 PMD profile of transmission span (solid) and perfect equalizer (dashed, dotted) in the 3-dimensional normalized Stokes or PMD vector space. © 1999 IEEE.

In a particular equalizer implementation EQ' may consist of the same sequence of SBAs and DGD sections as the TS, but with reversed order. The signs of SBA and PS retardations are inverted which can also be accomplished by orthogonal orientations.

$$\mathrm{EQ}' = \prod_{i=1}^{N} \left(\mathrm{PS}(-\omega\tau_i)\mathrm{SBA}\left(-\varphi_{2,i}, \varphi_{3i} + \sum_{k=i+1}^{N}(\varphi_{1k} + \varphi_{3k})\right)\right) \tag{2.440}$$

The phase delay $-\omega\tau_i$ of a DGD section may be of the order of many, many 2π. It is therefore practically impossible to avoid frequency-independent, possibly endless offset retardations $\varphi_{4i} = -\infty...\infty$ in each DGD section. However, these can be taken care of by moving them through subsequent SBAs, i.e. to the left, according to (2.437). In this generalized implementation with additional degrees-of-freedom the equalizer (EQ) is described by

$$\begin{aligned} \mathrm{EQ} &= \mathrm{PS}\left(\sum_{k=1}^{N}\varphi_{4k}\right)\mathrm{EQ}' \\ &= \prod_{i=1}^{N}\left(\mathrm{PS}(-\omega\tau_i + \varphi_{4i})\mathrm{SBA}\left(-\varphi_{2i}, \varphi_{3i} + \sum_{k=i+1}^{N}(\varphi_{4k} + \varphi_{1k} + \varphi_{3k})\right)\right) \end{aligned}. \tag{2.441}$$

As desired, the concatenation of TS and EQ (or EQ') results in a frequency-independent ER that does not exhibit any PMD,

$$\mathrm{EQ} \cdot \mathrm{TS} = \mathrm{PS}\left(\sum_{i=1}^{N}(\varphi_{4i} + \varphi_{1i} + \varphi_{3i})\right)\mathrm{ER}_0 = \mathrm{ER}\,. \tag{2.442}$$

Although suggested by (2.441), the polarization transformers in the EQ need not necessarily be SBAs. We may substitute $\varphi_{4i} = \varphi'_{4i} + \varphi''_{4i}$ and consider $\mathrm{PS}(\varphi'_{4i})$ to be part of a DGD section $\mathrm{PS}(-\omega\tau_i + \varphi'_{4i})$ while $\mathrm{PS}(\varphi''_{4i})$ belongs to the preceding polarization transformer. The *i*th polarization transformer, now described by

$$\mathrm{PS}(\varphi''_{4i})\mathrm{SBA}\left(-\varphi_{2i}, \varphi_{3i} + \sum_{k=i+1}^{N}(\varphi'_{4k} + \varphi''_{4k} + \varphi_{1k} + \varphi_{3k})\right), \tag{2.443}$$

is a general ER (2.437) if its three variables, one for the PS and two for the SBA, are independent. However, to minimize efforts the designer should let φ''_{4i} depend on the two SBA parameters at will, thereby retaining only two independent variables. Such a polarization transformer may sometimes be easier to realize than a true SBA. Like the SBA it has the property of being able to **endlessly transform any input polarization into a PSP of the following DGD section**. Firstly, *any* polarization transformer capable of such operation may be used here, since the PS in (2.443) constitutes the difference between the SBA, which is needed to fulfill the required functionality, and the most general case, an ER. Secondly, it is indeed a PSP we need to consider here, not just an eigenmode. To see this we place another, arbitrary retarder and its inverse between $\mathrm{PS}(\varphi''_{4i})$ and $\mathrm{PS}(-\omega\tau_i+\varphi'_{4i})$. One of them is considered to belong to $\mathrm{PS}(-\omega\tau_i+\varphi'_{4i})$ where it transforms PSPs (but modifies EMs in a different way), while the other is part of the polarization transformer (2.443) and will transform horizontal and vertical polarizations which are available at the output of $\mathrm{PS}(\varphi''_{4i})$ into the transformed PSPs.

Eqn. (2.443) can also be implemented by a phase shifter placed between two mode converters, all with finite retardations [11]. The matrix $\mathrm{MC}(0...\pi)\mathrm{PS}(0...2\pi)\mathrm{MC}(0...\pi)$ is a suitable description. In this particular example, which requires a proper control algrorithm, φ''_{4i} in the equivalent expression (2.443) is sometimes a step function of the SBA orientation angle. This matter can complicate control considerably because φ''_{4i} appears in the orientation angles of all subsequent SBAs of the PMD compensator. On the other hand, practical difficulties are minimized if φ''_{4i} can be chosen to be constant.

Problem: A reciprocal polarization transformer which can be represented by the matrix $\mathrm{PS}(\psi_3)\mathrm{SBA}(\psi_1,\psi_2)$ with $\psi_3=f(\psi_1,\psi_2)$ can in forward direction endlessly transform any input polarization into x or y output polarization. (1) Show that in backward direction it can endlessly transform x or y polarization into any polarization. (2) Under which condition can it, in forward direction, also endlessly transform x or y input polarization into any output polarization? (3) A reciprocal polarization transformer is now given by $\mathrm{MC}(\delta_2)\mathrm{PS}(\delta_1)$. Can you represent it as $\mathrm{PS}(\psi_3)\mathrm{SBA}(\psi_1,\psi_2)$ with $\psi_3=f(\psi_1,\psi_2)$? (4) Can it in backward direction endlessly transform x or y polarization into any polarization? (5) Can it, in forward direction, endlessly transform x or y input polarization into any output polarization?

Solution: In the following we refer to Jones matrices. For reciprocity the Jones matrix in backward direction is the transpose of that in forward direction.

(1) For the backward direction we obtain $(\mathrm{PS}(\psi_3)\mathrm{SBA}(\psi_1,\psi_2))^T=\mathrm{SBA}(\psi_1,\psi_2)^T\,\mathrm{PS}(\psi_3)^T$ $=\mathrm{SBA}(\psi_1,-\psi_2)\mathrm{PS}(\psi_3)$; see also (2.252). $\mathrm{PS}(\psi_3)$ leaves x and y polarizations unchanged, and $\mathrm{SBA}(\psi_1,\psi_4)$ with $\psi_4=-\psi_2=-\infty...\infty$ has all necessary properties to be able to endlessly transform this x or y polarization into any polarization. Alternative, more complicated proof: The inverse of $\mathrm{SBA}(\psi_1,-\psi_2)\mathrm{PS}(\psi_3)$ must be able to endlessly transform any polarization into x or y polarization. That is indeed the case because we can write $(\mathrm{SBA}(\psi_1,-\psi_2)\mathrm{PS}(\psi_3))^{-1}=\mathrm{PS}(\psi_3)^{-1}\mathrm{SBA}(\psi_1,-\psi_2)^{-1}=\mathrm{PS}(-\psi_3)\mathrm{SBA}(-\psi_1,-\psi_2)$ $=\mathrm{PS}(\psi_6)\mathrm{SBA}(\psi_1,\psi_5)$ with $\psi_5=\pi-\psi_2=-\infty...\infty$ and $\psi_6=-\psi_3=g(\psi_1,\psi_5)$.

(2) With the assumed reciprocity the task is identical to endlessly transforming the arbitrary polarization of a backward-propagating signal at the output into x or y polarization at the input. The Jones matrix in backward direction $\mathrm{SBA}(\psi_1,-\psi_2)\mathrm{PS}(\psi_3)$ can according to (2.437) be written as $\mathrm{PS}(\psi_3)\mathrm{SBA}(\psi_1,-\psi_2-\psi_3)$. The task is fulfilled if $\mathrm{SBA}(\psi_1,\psi_7)$ with $\psi_7=-\psi_2-\psi_3$ is an SBA. So the question is whether $\psi_7=-\psi_2-f(\psi_1,\psi_2)$ can be continuously and strictly

monotonically varied in the range $-\infty...+\infty$ by varying the only available independent variable ψ_2. It must therefore hold $0<\partial\psi_7/\partial\psi_2<\infty$ or $-\infty<\partial\psi_7/\partial\psi_2<0$. Using $\partial\psi_7/\partial\psi_2=-1-\partial\psi_3/\partial\psi_2$ it follows $-\infty<\partial\psi_3/\partial\psi_2<-1$ or $-1<\partial\psi_3/\partial\psi_2<\infty$. The second expression is more easily fulfilled than the first, in particular by $\psi_3 = \text{const.}$.

(3) It holds $\mathrm{PS}(\psi_3)\mathrm{SBA}(\psi_1,\psi_2) = \begin{bmatrix} e^{j\psi_3/2}\cos\psi_1/2 & je^{j(\psi_2+\psi_3/2)}\sin\psi_1/2 \\ je^{-j(\psi_2+\psi_3/2)}\sin\psi_1/2 & e^{-j\psi_3/2}\cos\psi_1/2 \end{bmatrix}$ and $\mathrm{MC}(\delta_2)\mathrm{PS}(\delta_1) = \begin{bmatrix} e^{j\delta_1/2}\cos\delta_2/2 & je^{-j\delta_1/2}\sin\delta_2/2 \\ je^{j\delta_1/2}\sin\delta_2/2 & e^{-j\delta_1/2}\cos\delta_2/2 \end{bmatrix}$. An element comparison yields successively $\delta_2=\psi_1$, $\psi_3/2=\delta_1/2=-\psi_2-\psi_3/2$, $\psi_3=-\psi_2$, $\delta_1=-\psi_2$. Yes, the required δ_1, δ_2, ψ_3 are indeed given as functions of ψ_1, ψ_2. The device can in forward direction endlessly transform any input polarization into x or y output polarization. Much easier to understand, $\mathrm{PS}(\delta_1)$ transforms the input polarization to a position on the S_2 - S_3 great circle from where the subsequent $\mathrm{MC}(\delta_2)$ can transform it into x and y polarization.

(4) We combine solutions (3) and (1) and find: yes.

(5) We need to fulfill either $-\infty<\partial\psi_3/\partial\psi_2<-1$ or $-1<\partial\psi_3/\partial\psi_2<\infty$. But in (3) we have found $\psi_3=-\psi_2$, which means $\partial\psi_3/\partial\psi_2=-1$. The answer is no. Much easier to understand, $\mathrm{PS}(\delta_1)$ leaves x and y input polarizations unchanged, and the subsequent $\mathrm{MC}(\delta_2)$ can therefore reach only half the S_2 - S_3 great circle but not the whole Poincaré sphere.

Fig. 2.30 shows the DGD profile of the concatenation of TS and a perfect EQ fulfilling (2.442). All PMD vectors are canceled by opposed adjacent ones because PMD is compensated not just to 1st order (which merely requires the vector sum and thereby the overall PMD vector to vanish) but completely (assuming that all existing frequency dependence has been covered by vectors Ω_i). As exemplified by the dotted arrows, an excess of total DGD of the EQ over that of the TS is not of concern if some adjacent compensator sections are made to cancel each other. Less perfect PMD vector cancelling normally indicates a PMD penalty. Provided the signal happens to coincide with a PSP it may be transmitted without distortion if merely 1st-order PMD persists after the EQ.

An important practical question is whether a PMD compensator should have fixed or variable DGD sections, and there is a straight answer to it [10]. It may be calculated that fixed differential group delays can cause detrimental side maxima of compensator performance. An immediate argument in favor of a variable DGD compensator is therefore that side maxima vanish. This is useful and important for one-section equalizers which, however, leave 2nd- and higher-order PMD uncompensated.

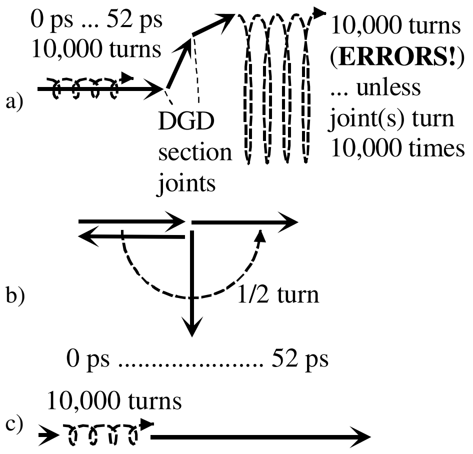


Fig. 2.31 Variable DGD section equalizer has speed problem (a). Transmission span most easily changes PMD vector orientations, and so does a fixed DGD section equalizer (b). Pure DGD vector lengthening is unlikely to happen in a transmission span (c). © 1999 IEEE.

The situation turns out to be different for equalizers with more than one section: What happens if the DGD of the first section (denoted as $-\tilde{\mathbf{\Omega}}_N$ in Fig. 2.30) of a two- or even multisection EQ with *variable* DGDs has to slide from 0 to 52 ps? The latter value corresponds to roughly 10,000 periods of a 1550 nm lightwave. The DGD increase corresponds to a lengthening of this vector by a screw motion with a pitch of one lightwave period of DGD change per turn. If the subsequent DGD vectors (denominated as $-\tilde{\mathbf{\Omega}}_{N-1}$ ... $-\tilde{\mathbf{\Omega}}_1$) are connected to it by fixed joints (= SBAs with frozen parameters) the PMD profile will change shape during each screw turn, which is highly detrimental (Fig. 2.31a). If the joints are of rotary type (= SBAs with variable orientations) the PMD profile may stay the same because all subsequent DGD vectors may revolve in place 10,000 times like axes connected by rotary joints. If the first section is followed by a ball-and-socket joint (= ER with 3 degrees-of-freedom) only this latter has to turn 10,000 times. The issue can also be understood from $\tilde{\mathbf{\Omega}}_i = \left(\prod_{j=1}^{i-1} \mathbf{G}_j^T\right)\mathbf{\Omega}_i$. Now, if we consider that each of the 10,000 turns of the joint(s) (SBAs or ER) may need between 10 and 100 optimization steps it becomes clear that variable DGD sections are unpractical due to the huge required number of SBA (or ER) adjustment steps, except for the last section which has no subsequent SBA. For comparison, consider two *fixed* 26 ps DGD sections in the compensator. Changing the SBA (or ER) in between these two by a retardation of just π will flip the DGD profile open like a pocketknife from 0 ps to a total DGD of 52 ps (Fig. 2.31b), and this is about 10,000 times faster than the previous case.

But could a fixed DGD equalizer follow if a TS chose to vary its PMD profile by lengthening a DGD vector? Here we explain why substantial DGD vector length changes are unlikely to happen: Consider a TS with two *fixed* 26 ps DGD sections. As already explained, a DGD change of 52 ps requires just a retardation change of π for the SBA (or ER) in between them (Fig. 2.31b). In contrast, a pure 52 ps DGD *growth* (PMD vector lengthening) requires a much higher retardation change of 10,000·2π and therefore occurs with a negligibly small probability (Fig. 2.31c).

As a consequence, **an equalizer with fixed DGD sections is a natural PMD compensator for a fiber transmission span** whereas more than one variable DGD section of an equalizer can practically not be used as such. Nevertheless, a single-section variable DGD equalizer is able to compensate 1st-order PMD, better than a single-section fixed DGD equalizer.

According to what we have learnt the DGD profile of the TE-TM converter of Fig. 2.25 is a straight arrow or rod as long as there is no mode conversion. This rod can be bent in any direction by a proper combination of in-phase and quadrature mode conversion. Twisting about the axis will only occur as a function of optical frequency but not electrooptically. The DGD profile of, say, Fig. 2.30 is just an approximation. In reality it is more likely that the DGD profile of the fiber will be smooth, without sharp corners. It is only logical to construct a near-perfect PMD equalizer from a sufficiently large number of such TE-TM mode converters on one chip. The DGD is about 0.26ps/mm. With possible chip sizes of almost 100 mm this is sufficient to compensate one bit duration of DGD at 40 Gbit/s. The DGD profile of the equalizer will be a bend-flexible but torsion-stiff rod, with close to ideal properties. Fig. 2.32 shows a small section of such a distributed PMD equalizer [18]. See also [24, 25] for further discussion on PMD and its compensation.

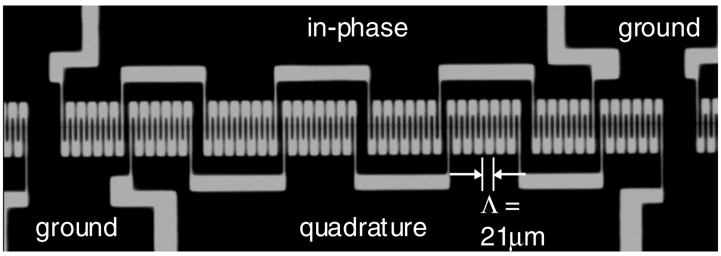


Fig. 2.32 One out of 73 TE-TM mode converters on an X-cut, Y-propagation $LiNbO_3$ chip. The waveguide runs underneath the comb electrodes.

2.6.3 Chromatic Dispersion Compensation

The Jones matrix (2.421) describes a birefringent waveguide with periodic mode coupling. We design it to have a length equal to an integer number m of coupling periods, $\beta_p z = 2\pi m$. The Jones matrix is now