Contents

List of Figures xiii

List of Tables xvii

List of Examples xix

Foreword xxi

Preface to Volume III xxv

III.1 Bonds and Swaps 1
- III.1.1 Introduction 1
- III.1.2 Interest Rates 2
  - III.1.2.1 Continuously Compounded Spot and Forward Rates 3
  - III.1.2.2 Discretely Compounded Spot Rates 4
  - III.1.2.3 Translation between Discrete Rates and Continuous Rates 6
  - III.1.2.4 Spot and Forward Rates with Discrete Compounding 6
  - III.1.2.5 LIBOR 8
- III.1.3 Categorization of Bonds 8
  - III.1.3.1 Categorization by Issuer 9
  - III.1.3.2 Categorization by Coupon and Maturity 10
- III.1.4 Characteristics of Bonds and Interest Rates 10
  - III.1.4.1 Present Value, Price and Yield 11
  - III.1.4.2 Relationship between Price and Yield 13
  - III.1.4.3 Yield Curves 14
  - III.1.4.4 Behaviour of Market Interest Rates 17
  - III.1.4.5 Characteristics of Spot and Forward Term Structures 19
- III.1.5 Duration and Convexity 20
  - III.1.5.1 Macaulay Duration 21
  - III.1.5.2 Modified Duration 23
  - III.1.5.3 Convexity 24
  - III.1.5.4 Duration and Convexity of a Bond Portfolio 24
  - III.1.5.5 Duration–Convexity Approximations to Bond Price Change 25
  - III.1.5.6 Immunizing Bond Portfolios 26

III.1.6 Bonds with Semi-Annual and Floating Coupons 28
  III.1.6.1 Semi-Annual and Quarterly Coupons 29
  III.1.6.2 Floating Rate Notes 31
  III.1.6.3 Other Floaters 33
III.1.7 Forward Rate Agreements and Interest Rate Swaps 33
  III.1.7.1 Forward Rate Agreements 34
  III.1.7.2 Interest Rate Swaps 35
  III.1.7.3 Cash Flows on Vanilla Swaps 36
  III.1.7.4 Cross-Currency Swaps 38
  III.1.7.5 Other Swaps 40
III.1.8 Present Value of a Basis Point 41
  III.1.8.1 PV01 and Value Duration 41
  III.1.8.2 Approximations to PV01 44
  III.1.8.3 Understanding Interest Rate Risk 45
III.1.9 Yield Curve Fitting 48
  III.1.9.1 Calibration Instruments 48
  III.1.9.2 Bootstrapping 49
  III.1.9.3 Splines 51
  III.1.9.4 Parametric Models 52
  III.1.9.5 Case Study: Statistical Properties of Forward LIBOR Rates 53
III.1.10 Convertible Bonds 59
  III.1.10.1 Characteristics of Convertible Bonds 60
  III.1.10.2 Survey of Pricing Models for Convertible Bonds 61
III.1.11 Summary and Conclusions 62

III.2 Futures and Forwards 65
III.2.1 Introduction 65
III.2.2 Characteristics of Futures and Forwards 68
  III.2.2.1 Interest Rate and Swap Futures 68
  III.2.2.2 Bond Futures 70
  III.2.2.3 Currency Futures and Forwards 73
  III.2.2.4 Energy and Commodity Futures 74
  III.2.2.5 Stock Futures and Index Futures 79
  III.2.2.6 Exchange Traded Funds and ETF Futures 80
  III.2.2.7 New Futures Markets 82
III.2.3 Theoretical Relationships between Spot, Forward and Futures 87
  III.2.3.1 No Arbitrage Pricing 87
  III.2.3.2 Accounting for Dividends 88
  III.2.3.3 Dividend Risk and Interest Rate Risk 90
  III.2.3.4 Currency Forwards and the Interest Rate Differential 91
  III.2.3.5 No Arbitrage Prices for Forwards on Bonds 92
  III.2.3.6 Commodity Forwards, Carry Costs and Convenience Yields 93
  III.2.3.7 Fair Values of Futures and Spot 94
III.2.4 The Basis 95
  III.2.4.1 No Arbitrage Range 95

- III.2.4.2 Correlation between Spot and Futures Returns 97
- III.2.4.3 Introducing Basis Risk 98
- III.2.4.4 Basis Risk in Commodity Markets 100

III.2.5 Hedging with Forwards and Futures 101
- III.2.5.1 Traditional 'Insurance' Approach 102
- III.2.5.2 Mean–Variance Approach 104
- III.2.5.3 Understanding the Minimum Variance Hedge Ratio 106
- III.2.5.4 Position Risk 108
- III.2.5.5 Proxy Hedging 110
- III.2.5.6 Basket Hedging 111
- III.2.5.7 Performance Measures for Hedged Portfolios 112

III.2.6 Hedging in Practice 113
- III.2.6.1 Hedging Forex Risk 113
- III.2.6.2 Hedging International Stock Portfolios 114
- III.2.6.3 Case Study: Hedging an Energy Futures Portfolio 118
- III.2.6.4 Hedging Bond Portfolios 124

III.2.7 Using Futures for Short Term Hedging 126
- III.2.7.1 Regression Based Minimum Variance Hedge Ratios 127
- III.2.7.2 Academic Literature on Minimum Variance Hedging 129
- III.2.7.3 Short Term Hedging in Liquid Markets 131

III.2.8 Summary and Conclusions 133

III.3 Options 137

III.3.1 Introduction 137

III.3.2 Foundations 139
- III.3.2.1 Arithmetic and Geometric Brownian Motion 140
- III.3.2.2 Risk Neutral Valuation 142
- III.3.2.3 Numeraire and Measure 144
- III.3.2.4 Market Prices and Model Prices 146
- III.3.2.5 Parameters and Calibration 147
- III.3.2.6 Option Pricing: Review of the Binomial Model 148

III.3.3 Characteristics of Vanilla Options 151
- III.3.3.1 Elementary Options 152
- III.3.3.2 Put–Call Parity 153
- III.3.3.3 Moneyness 154
- III.3.3.4 American Options 155
- III.3.3.5 Early Exercise Boundary 156
- III.3.3.6 Pricing American Options 158

III.3.4 Hedging Options 159
- III.3.4.1 Delta 159
- III.3.4.2 Delta Hedging 161
- III.3.4.3 Other Greeks 161
- III.3.4.4 Position Greeks 163
- III.3.4.5 Delta–Gamma Hedging 164
- III.3.4.6 Delta–Gamma–Vega Hedging 165

III.3.5 Trading Options 167
- III.3.5.1 Bull Strategies 167

- III.3.5.2 Bear Strategies 168
- III.3.5.3 Other Spread Strategies 169
- III.3.5.4 Volatility Strategies 170
- III.3.5.5 Replication of P&L Profiles 172
- III.3.6 The Black–Scholes–Merton Model 173
  - III.3.6.1 Assumptions 174
  - III.3.6.2 Black–Scholes–Merton PDE 175
  - III.3.6.3 Is the Underlying the Spot or the Futures Contract? 176
  - III.3.6.4 Black–Scholes–Merton Pricing Formula 178
  - III.3.6.5 Interpretation of the Black–Scholes–Merton Formula 180
  - III.3.6.6 Implied Volatility 183
  - III.3.6.7 Adjusting BSM Prices for Stochastic Volatility 183
- III.3.7 The Black–Scholes–Merton Greeks 186
  - III.3.7.1 Delta 187
  - III.3.7.2 Theta and Rho 188
  - III.3.7.3 Gamma 189
  - III.3.7.4 Vega, Vanna and Volga 190
  - III.3.7.5 Static Hedges for Standard European Options 193
- III.3.8 Interest Rate Options 194
  - III.3.8.1 Caplets and Floorlets 195
  - III.3.8.2 Caps, Floors and their Implied Volatilities 196
  - III.3.8.3 European Swaptions 198
  - III.3.8.4 Short Rate Models 199
  - III.3.8.5 LIBOR Model 201
  - III.3.8.6 Case Study: Application of PCA to LIBOR Model Calibration 203
- III.3.9 Pricing Exotic Options 207
  - III.3.9.1 Pay-offs to Exotic Options 208
  - III.3.9.2 Exchange Options and Best/Worst of Two Asset Options 209
  - III.3.9.3 Spread Options 211
  - III.3.9.4 Currency Protected Options 213
  - III.3.9.5 Power Options 214
  - III.3.9.6 Chooser Options and Contingent Options 214
  - III.3.9.7 Compound Options 216
  - III.3.9.8 Capped Options and Ladder Options 216
  - III.3.9.9 Look-Back and Look-Forward Options 218
  - III.3.9.10 Barrier Options 219
  - III.3.9.11 Asian Options 221
- III.3.10 Summary and Conclusions 224

III.4 Volatility **227**
- III.4.1 Introduction 227
- III.4.2 Implied Volatility 231
  - III.4.2.1 'Backing Out' Implied Volatility from a Market Price 231
  - III.4.2.2 Equity Index Volatility Skew 233
  - III.4.2.3 Smiles and Skews in Other Markets 236

III.4.2.4 Term Structures of Implied Volatilities 238
III.4.2.5 Implied Volatility Surfaces 239
III.4.2.6 Cap and Caplet Volatilities 240
III.4.2.7 Swaption Volatilities 242
III.4.3 Local Volatility 243
III.4.3.1 Forward Volatility 244
III.4.3.2 Dupire's Equation 245
III.4.3.3 Parametric Models of Local Volatility 248
III.4.3.4 Lognormal Mixture Diffusion 249
III.4.4 Modelling the Dynamics of Implied Volatility 255
III.4.4.1 Sticky Models 255
III.4.4.2 Case Study I: Principal Component Analysis of Implied Volatilities 257
III.4.4.3 Case Study II: Modelling the ATM Volatility–Index Relationship 261
III.4.4.4 Case Study III: Modelling the Skew Sensitivities 264
III.4.4.5 Applications of Implied Volatility Dynamics to Hedging Options 265
III.4.5 Stochastic Volatility Models 268
III.4.5.1 Stochastic Volatility PDE 269
III.4.5.2 Properties of Stochastic Volatility 271
III.4.5.3 Model Implied Volatility Surface 275
III.4.5.4 Model Local Volatility Surface 277
III.4.5.5 Heston Model 278
III.4.5.6 GARCH Diffusions 280
III.4.5.7 CEV and SABR Models 285
III.4.5.8 Jumps in Prices and in Stochastic Volatility 287
III.4.6 Scale Invariance and Hedging 289
III.4.6.1 Scale Invariance and Change of Numeraire 291
III.4.6.2 Definition of Scale Invariance 291
III.4.6.3 Scale Invariance and Homogeneity 292
III.4.6.4 Model Free Price Hedge Ratios 294
III.4.6.5 Minimum Variance Hedging 297
III.4.6.6 Minimum Variance Hedge Ratios in Specific Models 299
III.4.6.7 Empirical Results 300
III.4.7 Trading Volatility 303
III.4.7.1 Variance Swaps and Volatility Swaps 304
III.4.7.2 Trading Forward Volatility 306
III.4.7.3 Variance Risk Premium 307
III.4.7.4 Construction of a Volatility Index 308
III.4.7.5 Effect of the Skew 309
III.4.7.6 Term Structures of Volatility Indices 309
III.4.7.7 Vix and Other Volatility Indices 311
III.4.7.8 Volatility Index Futures 312
III.4.7.9 Options on Volatility Indices 314
III.4.7.10 Using Realized Volatility Forecasts to Trade Volatility 315
III.4.8 Summary and Conclusion 316

III.5 Portfolio Mapping **321**
III.5.1 Introduction 321
III.5.2 Risk Factors and Risk Factor Sensitivities 323
III.5.2.1 Interest Rate Sensitive Portfolios 323
III.5.2.2 Equity Portfolios 324
III.5.2.3 International Exposures 327
III.5.2.4 Commodity Portfolios 328
III.5.2.5 Option Portfolios 328
III.5.2.6 Orthogonalization of Risk Factors 330
III.5.2.7 Nominal versus Percentage Risk Factors and Sensitivities 330
III.5.3 Cash Flow Mapping 332
III.5.3.1 Present Value Invariant and Duration Invariant Maps 332
III.5.3.2 PV01 Invariant Cash Flow Maps 333
III.5.3.3 Volatility Invariant Maps 334
III.5.3.4 Complex Cash Flow Maps 336
III.5.4 Applications of Cash Flow Mapping to Market Risk Management 337
III.5.4.1 Risk Management of Interest Rate Sensitive Portfolios 337
III.5.4.2 Mapping Portfolios of Commodity Futures 338
III.5.5 Mapping an Option Portfolio to Price Risk Factors 340
III.5.5.1 Taylor Expansions 341
III.5.5.2 Value Delta and Value Gamma 342
III.5.5.3 Delta–Gamma Approximation: Single Underlying 344
III.5.5.4 Effect of Gamma on Portfolio Risk 346
III.5.5.5 Price Beta Mapping 347
III.5.5.6 Delta–Gamma Approximation: Several Underlyings 349
III.5.5.7 Including Time and Interest Rates Sensitivities 351
III.5.6 Mapping Implied Volatility 353
III.5.6.1 Vega Risk in Option Portfolios 353
III.5.6.2 Second Order Approximations: Vanna and Volga 354
III.5.6.3 Vega Bucketing 355
III.5.6.4 Volatility Beta Mapping 356
III.5.7 Case Study: Volatility Risk in FTSE 100 Options 357
III.5.7.1 Estimating the Volatility Betas 357
III.5.7.2 Model Risk of Volatility Mapping 360
III.5.7.3 Mapping to Term Structures of Volatility Indices 361
III.5.7.4 Using PCA with Volatility Betas 361
III.5.8 Summary and Conclusions 364

References **367**

Index **377**