

Contents

Preface	xiii
1 Introduction to Nonlinear Programming	1
1.1 Preliminaries	1
1.2 Newton's Method in One Variable	2
1.3 Secant Method in One Variable	4
1.4 Newton's Method for Minimization in One Variable	5
1.5 Newton's Method in Several Variables	7
1.6 Unconstrained Optimization	8
1.7 Recursive Updates	10
1.8 Equality-Constrained Optimization	12
1.8.1 Newton's Method	15
1.9 Inequality-Constrained Optimization	16
1.10 Quadratic Programming	18
1.11 Globalization Strategies	21
1.11.1 Merit Functions	21
1.11.2 Line-Search Methods	23
1.11.3 Trust-Region Methods	25
1.11.4 Filters	26
1.12 Nonlinear Programming	28
1.13 An SQP Algorithm	29
1.14 Interior-Point Methods	30
1.15 Mathematical Program with Complementarity Conditions	36
1.15.1 The Signum or Sign Operator	37
1.15.2 The Absolute Value Operator	38
1.15.3 The Maximum Value Operator	38
1.15.4 The Minimum Value Operator	39
1.15.5 Solving an MPEC	39
1.16 What Can Go Wrong	40
1.16.1 Infeasible Constraints	40
1.16.2 Rank-Deficient Constraints	40
1.16.3 Constraint Redundancy	41
1.16.4 Discontinuities	42
1.16.5 Scaling	45
1.16.6 Nonunique Solution	46

1.17	Derivative Approximation by Finite Differences	46
1.17.1	Difference Estimates in Differential Equations	48
2	Large, Sparse Nonlinear Programming	51
2.1	Overview: Large, Sparse NLP Issues	51
2.2	Sparse Finite Differences	52
2.2.1	Background	52
2.2.2	Sparse Hessian Using Gradient Differences	53
2.2.3	Sparse Differences in Nonlinear Programming	54
2.3	Sparse QP Subproblem	54
2.4	Merit Function	56
2.5	Hessian Approximation	58
2.6	Sparse SQP Algorithm	60
2.6.1	Minimization Process	60
2.6.2	Algorithm Strategy	62
2.7	Defective Subproblems	62
2.8	Feasible Point Strategy	63
2.8.1	QP Subproblem	63
2.8.2	Feasible Point Strategy	64
2.8.3	An Illustration	65
2.9	Computational Experience	67
2.9.1	Large, Sparse Test Problems	67
2.9.2	Small, Dense Test Problems	68
2.10	Nonlinear Least Squares	70
2.10.1	Background	70
2.10.2	Sparse Least Squares	70
2.10.3	Residual Hessian	72
2.11	Barrier Algorithm	73
2.11.1	External Format	73
2.11.2	Internal Format	74
2.11.3	Definitions	76
2.11.4	Logarithmic Barrier Function	77
2.11.5	Computing a Search Direction	79
2.11.6	Inertia Requirements for the Barrier KKT System	82
2.11.7	Filter Globalization	83
2.11.8	Barrier Parameter Update Strategy	86
2.11.9	Initialization	86
2.11.10	Outline of the Primary Algorithm	87
2.11.11	Computational Experience	88
3	Optimal Control Preliminaries	91
3.1	The Transcription Method	91
3.2	Dynamic Systems	91
3.3	Shooting Method	93
3.4	Multiple Shooting Method	95
3.5	Initial Value Problems	97
3.6	Boundary Value Example	105

3.7	Dynamic Modeling Hierarchy	108
3.8	Function Generator	108
3.8.1	Description	108
3.8.2	NLP Considerations	109
3.9	Dynamic System Differentiation	111
3.9.1	Simple Example	111
3.9.2	Discretization versus Differentiation	115
3.9.3	External and Internal Differentiation	115
3.9.4	Variational Derivatives	118
4	The Optimal Control Problem	123
4.1	Introduction	123
4.1.1	Dynamic Constraints	123
4.1.2	Algebraic Equality Constraints	124
4.1.3	Singular Arcs	125
4.1.4	Algebraic Inequality Constraints	126
4.2	Necessary Conditions for the Discrete Problem	126
4.3	Direct versus Indirect Methods	127
4.4	General Formulation	129
4.5	Direct Transcription Formulation	132
4.6	NLP Considerations—Sparsity	134
4.6.1	Background	134
4.6.2	Standard Approach	136
4.6.3	Discretization Separability	137
4.6.4	Right-Hand-Side Sparsity (Trapezoidal)	139
4.6.5	Hermite–Simpson (Compressed) (HSC)	141
4.6.6	Hermite–Simpson (Separated) (HSS)	143
4.6.7	K-Stage Runge–Kutta Schemes	145
4.6.8	General Approach	146
4.6.9	Performance Issues	147
4.6.10	Performance Highlights	149
4.7	Mesh Refinement	152
4.7.1	Representing the Solution	153
4.7.2	Estimating the Discretization Error	154
4.7.3	Estimating the Order Reduction	158
4.7.4	Constructing a New Mesh	159
4.7.5	The Mesh-Refinement Algorithm	161
4.7.6	Computational Experience	163
4.8	Scaling	166
4.9	Quadrature Equations	168
4.10	Algebraic Variable Rate Constraints	172
4.11	Estimating Adjoint Variables	173
4.11.1	Quadrature Approximation	175
4.11.2	Path Constraint Adjoints	176
4.11.3	Differential Constraint Adjoints	177
4.11.4	Numerical Comparisons	178
4.12	Discretize Then Optimize	192

4.12.1	High Index Partial Differential-Algebraic Equation	192
4.12.2	State Vector Formulation	193
4.12.3	Direct Transcription Results	194
4.12.4	The Indirect Approach	194
4.12.5	Optimality Conditions	196
	Unconstrained Arcs ($s < 0$)	196
	Constrained Arcs ($s = 0$)	197
	Boundary Conditions	198
	Optimality Conditions: Summary	199
4.12.6	Computational Comparison—Direct versus Indirect	199
	Direct Method	199
	Indirect Method	199
4.12.7	Analysis of Results	200
	The Quandary	200
	The Explanation	201
4.13	Questions of Efficiency	205
	Question: Newton or Quasi-Newton Hessian?	210
	Question: Barrier or SQP Algorithm?	210
4.14	What Can Go Wrong	212
4.14.1	Singular Arcs	212
4.14.2	State Constraints	215
4.14.3	Discontinuous Control	216
5	Parameter Estimation	219
5.1	Introduction	219
5.2	The Parameter Estimation Problem	219
5.3	Computing the Residuals	222
5.4	Computing Derivatives	223
5.4.1	Residuals and Sparsity	224
5.4.2	Residual Decomposition	225
5.4.3	Auxiliary Function Decomposition	225
5.4.4	Algebraic Variable Parameterization	227
5.5	Computational Experience	228
5.5.1	Reentry Trajectory Reconstruction	230
5.5.2	Commercial Aircraft Rotational Dynamics Analysis	233
5.6	Optimal Control or Optimal Estimation?	241
6	Optimal Control Examples	247
6.1	Space Shuttle Reentry Trajectory	247
6.2	Minimum Time to Climb	256
6.2.1	Tabular Data	257
6.2.2	Cubic Spline Interpolation	258
6.2.3	Minimum Curvature Spline	259
6.2.4	Numerical Solution	262
6.3	Low-Thrust Orbit Transfer	265
6.3.1	Modified Equinoctial Coordinates	265
6.3.2	Gravitational Disturbing Acceleration	267

6.3.3	Thrust Acceleration—Burn Arcs	267
6.3.4	Boundary Conditions	269
6.3.5	Numerical Solution	269
6.4	Two-Burn Orbit Transfer	271
6.4.1	Simple Shooting Formulation	273
6.4.2	Multiple Shooting Formulation	278
6.4.3	Collocation Formulation	279
6.5	Hang Glider	282
6.6	Abort Landing in the Presence of Windshear	284
6.6.1	Dynamic Equations	286
6.6.2	Objective Function	288
6.6.3	Control Variable	289
6.6.4	Model Data	289
6.6.5	Computational Results	291
6.7	Space Station Attitude Control	293
6.8	Reorientation of an Asymmetric Rigid Body	299
6.8.1	Computational Issues	300
6.9	Industrial Robot	304
6.10	Multibody Mechanism	310
6.11	Kinematic Chain	315
6.12	Dynamic MPEC	322
6.13	Free-Flying Robot	326
6.14	Kinetic Batch Reactor	331
6.15	Delta III Launch Vehicle	336
6.16	A Two-Strain Tuberculosis Model	345
6.17	Tumor Anti-angiogenesis	348
7	Advanced Applications	353
7.1	Optimal Lunar Swingby Trajectories	353
7.1.1	Background and Motivation	353
7.1.2	Optimal Lunar Transfer Examples	355
	Synchronous Equatorial	355
	Polar, 24 hr (A)	355
	Polar, 24 hr (B)	355
	Retrograde Molniya	357
7.1.3	Equations of Motion	357
7.1.4	Kepler Orbit Propagation	358
7.1.5	Differential-Algebraic Formulation of Three-Body Dynamics .	360
7.1.6	Boundary Conditions	360
7.1.7	A Four-Step Solution Technique	362
	Step 1: Three-Impulse, Conic Solution	362
	Step 2: Three-Body Approximation	364
	Step 3: Fixed Swingby Time	365
	Step 4: Optimal Three-Body Solution	366
7.1.8	Solving the Subproblems	366
	Is Mesh Refinement Needed?	368
	DAE or ODE Formulation?	370

7.2	Multiple-Pass Aero-Assisted Orbit Transfer	372
7.2.1	Orbital Phases	372
7.2.2	Atmospheric Phases	373
7.2.3	Boundary Conditions	375
7.2.4	Initial Guess	377
7.2.5	Numerical Results	379
7.3	Delay Differential Equations	385
7.4	In-Flight Dynamic Optimization of Wing Trailing Edge Surface Positions	396
7.4.1	Aircraft Dynamics for Drag Estimation	398
7.4.2	Step 1: Reference Trajectory Estimation	400
7.4.3	Step 2: Aerodynamic Drag Model Approximation	401
7.4.4	Step 3: Optimal Camber Prediction	402
7.4.5	Numerical Results	402
	777-200ER Flight Test	402
	Performance Comparison	403
8	Epilogue	411
Appendix: Software		413
A.1	Simplified Usage Dense NLP	413
A.2	Sparse NLP with Sparse Finite Differences	413
A.3	Optimal Control Using Sparse NLP	414
Bibliography		417
Index		431