Contents

1 The Quantum Mechanics of Solids 1
1.1 General Hamiltonian for a Solid 3
1.1.1 The Need for Approximations 4
1.1.2 Wave Functions and Pauli Exclusion Principle 7
1.2 Energy Bands 8
1.2.1 Tight-Binding Approximation 8
1.2.2 Linear Atomic Chain 11
1.2.3 Energy Bands Associated with Tightly Bound Atomic Levels 20
1.3 Summary 21

2 Crystalline Solids: Diffraction 23
2.1 Crystal Structures 25
2.1.1 Crystal Lattice 25
2.1.2 Two-Dimensional Crystals 27
2.1.3 Three-Dimensional Crystals 29
2.1.4 Beyond the Perfect Crystal 33
2.2 Diffraction 35
2.2.1 General Principles 35
2.2.2 Reciprocal Lattice 38
2.3 Determination of Crystal Structures 39
2.3.1 On the Bragg Diffraction Condition 39
2.3.2 Diffraction Intensity and Basis of the Primitive Cell 41
2.3.3 X-Ray Diffraction 42
2.4 Summary 44
2.5 Answers to Questions 45

3 Electronic Structure of Solids: Metals and Insulators 51
3.1 Electrons in a Periodic Potential 53
3.1.1 Bloch's Theorem in Three Dimensions 53
3.1.2 General Properties of Bloch States 55

Bibliografische Informationen
http://d-nb.info/100254002X

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

3.1.3 Electrons in a Weak Periodic Potential 56
3.1.4 First Brillouin Zone 59
3.2 Band Structure of Solids 60
3.2.1 Linear Chain 60
3.2.2 Band Structure of Two-Dimensional Solids 64
3.2.3 Three-Dimensional Band Structures 68
3.3 Metals, Insulators, and Semiconductors 72
3.3.1 Statistical Physics of Fermions 72
3.3.2 Real Solids: Conductor or Insulator? 74
3.3.3 Introduction to the Semiconductor Case 77
3.3.4 The Special Case of Graphene 78
3.3.5 Thermodynamic Properties of Metals 80
3.4 Experimental Determination of Band Structures 84
3.4.1 Optical Absorption in Insulators 84
3.4.2 Tunneling Effect and Scanning Tunneling Microscopy 85
3.4.3 Angle-Resolved Photoemission Spectroscopy 88
3.5 Summary 90
3.6 Answers to Questions 92

4 Electron Transport in Solids 99
4.1 Drude Model for Transport in an Electron Gas: Relaxation Time and Collisions 101
4.1.1 Electrical Conductivity 102
4.1.2 Thermal Conductivity and the Wiedemann–Franz Law 105
4.2 Electron Transport in a Fermion Gas 107
4.2.1 Electrical Conductivity 108
4.2.2 Thermal Conductivity and the Wiedemann–Franz Law 110
4.3 Electrons in a Lattice: Dynamics of Bloch Electrons 111
4.3.1 Group Velocity 111
4.3.2 Acceleration in Reciprocal Space and Real Space 113
4.3.3 Electronic Conductivity in a Crystal 115
4.4 Origin of Collisions 116
4.4.1 Experimental Observation 116
4.4.2 Scattering by Lattice Vibrations 118
4.4.3 Collisions with Impurities and Defects 119
4.5 Electrons, Holes, and Dopants in Semiconductors 121
4.6 Electrons, Holes, and Transport in Graphene 126
4.7 Summary 132
4.8 Answers to Questions 133

5 Introduction to Superconductivity 145
5.1 Conditions for Superconductivity 147
5.1.1 Persistent Currents 147
5.1.2 Critical Field and Critical Current 150

5.2 Difference Between a Perfect Conductor and a Superconductor: The Meissner Effect 152
5.2.1 Effect of a Magnetic Field on a Perfect Conductor 152
5.2.2 Meissner Effect: Exclusion of the Magnetic Field 154
5.2.3 London Equations and Penetration Depth 155
5.3 A Macroscopic Quantum Effect 158
5.3.1 Macroscopic Wave Function and Current 158
5.3.2 Quantisation of Magnetic Flux 159
5.3.3 Measuring the Flux Quantum 160
5.3.4 Josephson Effect 162
5.3.5 SQUID 165
5.4 Summary 167
5.5 Answers to Questions 168

6 Thermodynamics of Superconductors 175
6.1 Thermodynamics of Bulk Superconductors 177
6.1.1 Free Energy of the Superconducting State 178
6.1.2 Entropy and Specific Heat 179
6.2 Thin Films and Coherence Length 181
6.2.1 Thermodynamics of Thin Films 181
6.2.2 Coherence Length ξ 184
6.2.3 Mixed State: A Simple Model 185
6.3 Two Types of Superconductivity 188
6.3.1 Type I Superconductors 189
6.3.2 Simple London Model for a Vortex 189
6.3.3 Type II Superconductivity 191
6.3.4 Applications of Type II Superconductors 194
6.4 Summary 195
6.5 Answers to Questions 196

7 Microscopic Origins of Superconductivity 201
7.1 Conventional Metal Superconductors 203
7.1.1 Superconducting Metals and Alloys 203
7.1.2 Attractive Interaction Due to Phonons 205
7.1.3 The Band Gap in the Superconducting State 208
7.2 Cooper Instability 210
7.2.1 Electrons in the Conduction Band of a Semiconductor 212
7.2.2 Extra Electrons in a Metal 213
7.3 BCS Theory: Experimental Evidence 214
7.3.1 Ground State and Band Gap 215
7.3.2 Excited States in the BCS Theory 216
7.3.3 Electron Tunneling Determination of the Gap 217
7.3.4 Critical Temperature and BCS Theory 220
7.3.5 Coherence Length 221

7.4 High-T_c Superconductors 223
7.4.1 Cuprates 223
7.4.2 Other Families of Superconductors 225
7.5 Summary 226
7.6 Answers to Questions 227

8 Magnetism of Insulators 231
8.1 Magnetic Behaviour of Solids 234
8.2 Magnetism of Atoms 236
8.2.1 Hydrogen Atom 236
8.2.2 Multielectron Atoms with Filled Shells 237
8.2.3 An Atom with a Partially Filled Shell: Carbon 237
8.2.4 Atoms with Partially Filled Shells: Hund Rules 240
8.3 Paramagnetism of an Ensemble of Isolated Ions 243
8.4 Ordered Magnetic States 244
8.4.1 Interatomic Exchange Interaction 245
8.4.2 Heisenberg Model 247
8.4.3 Ferromagnetism: Molecular Field Approximation 249
8.5 Antiferromagnetism and Ferrimagnetism 252
8.6 From Insulator Magnetism to Metallic Magnetism 255
8.6.1 Mott–Hubbard Insulator 256
8.6.2 Mott Transition and Doped Mott–Hubbard Insulators 258
8.6.3 Magnetism and Superconductivity 259
8.6.4 Metallicity and Magnetism in a Band Approach 261
8.7 Summary 265
8.8 Answers to Questions 266

9 Magnetic Anisotropy, Domains, and Walls 269
9.1 Magnetic Anisotropy 272
9.1.1 Magnetocrystalline Anisotropy 272
9.1.2 Effect of Anisotropy: Irreversibility 273
9.2 Dipole Interactions, Demagnetising Fields, and Domains 277
9.2.1 Demagnetising Fields 277
9.2.2 Demagnetisation Energy and Magnetic Domains 282
9.3 Bloch Walls 285
9.4 Magnetic Hysteresis Cycles 288
9.4.1 Bitter Method for Observing Domains 288
9.4.2 Magnetisation Process and Hysteresis 289
9.5 Summary 291
9.6 Answers to Questions 292

10 Measurements in Magnetism: From the Macroscopic to the Microscopic Scale 295
10.1 Macroscopic Magnetic Measurements 297
10.1.1 Susceptibility or Magnetisation 298
10.1.2 Examples of Measurements on Superconductors 301

10.1.3 Hysteresimeters: Toroidal Geometry 303
10.2 Magnetic Surface Measurements and Magnetic Imaging 304
10.2.1 Microscale Measurements and Surface Imaging 305
10.2.2 Towards Submicron Scales 308
10.3 Summary 312
10.4 Answers to Questions 312

11 Spin Dynamics and Magnetic Resonance 317
11.1 Dynamics in Magnetism: General Considerations 319
11.1.1 Linear Response and Dissipation 320
11.1.2 Pulse Response and Frequency Response 321
11.2 Dynamics in Ferromagnets 322
11.2.1 Low Frequency Losses in Ferromagnets 322
11.2.2 Ferromagnetic Resonance 324
11.3 Resonance in the Paramagnetic Regime 331
11.3.1 Resonance for a Thermodynamic Ensemble of Spins 331
11.3.2 Nuclear Magnetic Resonance (NMR) 333
11.3.3 Spin Echoes: Transverse Relaxation 336
11.3.4 Applications 339
11.4 Summary 341
11.5 Answers to Questions 342

12 The Thermodynamics of Ferromagnets 345
12.1 Excited States and Low Temperature Properties 347
12.1.1 Magnons 347
12.1.2 Low Temperature Thermodynamic Properties 349
12.1.3 Experimental Detection of Magnons 351
12.2 The Magnetic Phase Transition 352
12.2.1 Mean Field Theory 352
12.2.2 Landau and Ginzburg–Landau Theories 354
12.2.3 Critical Behaviour 357
12.3 Summary 359
12.4 Answers to Questions 360

13 Problem Set 363
Problem 1 Debye–Waller Factor 367
Problem 2 Reflectance of Aluminium 373
Problem 3 Band Structure of $YBa_2Cu_3O_7$ 377
3.1 Isolated Copper–Oxygen Chain 378
3.2 Isolated Copper–Oxygen Plane 379
3.3 Chain and Plane 380
3.4 Realistic Models of $YBa_2Cu_3O_7$ 381
Problem 4 Electronic Energy and Stability of Alloys 391
Problem 5 Optical Response of Monovalent Metals 399
Problem 6 One-Dimensional TTF-TCNQ Compounds 405
6.1 Isolated Chains 405

6.2 Experimental Observations 406
6.3 Dimerised Chain 407
6.4 Peierls Transition 409
Problem 7 Insulator–Metal Transition 419
7.1 Tight-Binding Method for Hydrogen-Like Orbitals ... 419
7.2 Interactions Between Electrons 420
7.3 Alkali Elements and Hydrogen 423
7.4 Insulator–Metal Transition in Si–P 423
Problem 8 Cyclotron Resonance 441
8.1 Real and Reciprocal Space Paths of an Electron State . 441
8.2 A Semiconductor: Silicon 442
8.3 Metals 444
Problem 9 Phonons in Solids 453
9.1 Einstein Model 453
9.2 Debye Model 453
9.3 Experimental Detection of Phonons 455
9.4 Thermodynamic Properties 456
9.5 Resistivity 457
Problem 10 Thermodynamics of a Thin Superconducting Cylinder: Little–Parks Experiment 469
Problem 11 Direct and Alternating Josephson Effects in Zero Magnetic Field 481
11.1 Model Josephson Junction 481
11.2 Realistic Josephson Junction 481
11.3 Josephson Junction in a Microwave Field 483
Problem 12 Josephson Junction in a Magnetic Field 491
12.1 Current Distribution 492
12.2 Screening of the Magnetic Field 493
12.3 Josephson Plasma Resonance 495
Problem 13 Magnetisation of a Type II Superconductor 511
Problem 14 Electronic Structure and Superconductivity of V_3Si 523
Problem 15 Superconductivity of $NbSe_2$ 531
Problem 16 Magnesium Diboride: A New Superconductor? 541
16.1 Atomic and Electronic Structure of MgB_2 541
16.2 Superconductivity of MgB_2 543
Problem 17 Electronic Properties of La_2CuO_4 557
Problem 18 Properties of an Antiferromagnetic Solid 563
18.1 Preliminaries: The Ferromagnetic Case 563
18.2 Antiferromagnetic Transition 564
18.3 Susceptibility in the Antiferromagnetic State 565
Problem 19 Magnetism of Thin Films and Magneto-Optic Applications . 573
Problem 20 Magnetism of a Thin Film 579
20.1 Uniform Magnetisation 580
20.2 Non-uniform Situations 581
20.3 Detailed Investigation of Non-uniform Situations 583

Appendix A Physical Constants 593

Appendix B Some Useful Functions and Relations 595

Appendix C Standard Notation 599

Appendix D Specific Notation 601

References 607

Index 613