
Preface

Multibody dynamics analysis was originally developed as a tool for modeling rigid multibody systems with simple tree-like topologies, but has considerably evolved to the point where it can handle linearly and nonlinearly elastic multibody systems with arbitrary topologies. It is now used widely as a fundamental design tool in many areas of engineering.

This textbook has emerged over the past two decades from efforts to teach graduate courses in advanced dynamics and flexible multibody dynamics to engineering students. Although this book reviews the basic principles of dynamics, it is assumed that students enrolling in these graduate courses have completed a comprehensive set of undergraduate courses in statics, dynamics, deformable bodies, energy methods, and numerical analysis. The advanced dynamics course is, of course, a prerequisite for the flexible multibody dynamics course.

The book is divided into six parts. The first part presents the basic tools and concepts that form the foundation for the other parts. It begins with a review of basic operations on vectors and tensors. The second chapter deals with coordinate systems. The differential geometry of both curves and surfaces is presented and leads to path and surface coordinates. Chapter 3 reviews the basic principles of dynamics, starting with Newton's laws. The important concept of conservative forces is discussed. Systems of particles are then treated, leading to Euler's first and second laws.

Chapter 4 concludes the first part of the book with a detailed description of three-dimensional rotation. For most graduate students, this chapter is not really a review. Indeed, many undergraduate dynamics courses focus primarily on two-dimensional systems. Problems involving three-dimensional rotation, if treated at all, are often rushed in the last few weeks of the semester, leaving most students with insufficient time to absorb this difficult material.

Part 2 develops rigid body dynamics, the foundation of multibody dynamics. The analysis of the kinematics of rigid bodies is the focus of chapter 5. It starts with the analysis of the general displacement and velocity fields of a rigid body. The classical topics of relative velocities and accelerations are also addressed. The motion tensor and its properties are given an in-depth treatment.

Kinetics of rigid bodies is the focus of chapter 6. The various forms of Euler's law governing the rotational motion of rigid bodies are presented. While the emphasis of the chapter is on three-dimensional problems, planar motion is also treated in details.

Part 3 presents the fundamental concepts of analytical dynamics. Chapter 7 introduces the concepts of virtual displacement, virtual rotation, and virtual work. The principle of virtual work for static problems is given extensive coverage as this is an indispensable topic for the study of the variational and energy principles of dynamics presented in chapter 8. D'Alembert's principle, Hamilton's principle, and Lagrange's formulation are derived and their use illustrated with numerous examples.

Multibody systems are characterized by two distinguishing features: system components undergo finite relative rotations and these components are connected by mechanical joints that impose restrictions on their relative motion. The first distinguishing feature is of a purely kinematic nature: in multibody systems, overall and relative motions are finite, leading to inherently nonlinear problems. The second distinguishing feature is the main culprit for the complexity of many multibody formulations. Each component of a flexible multibody system is a *constrained dynamical system* because of the restrictions imposed on it by the mechanical joints connecting it to others.

The first three parts of the book present background material on *unconstrained dynamical systems*, *i.e.*, systems for which the number of generalized coordinates used to describe the system equals the number of degrees of freedom. In contrast, part 4 focuses on *constrained dynamical systems*. Chapter 9 presents Lagrange's multiplier technique and the distinction between holonomic and nonholonomic constraints. The combination of the principle of virtual work with Lagrange's multiplier technique is shown to provide a powerful tool for the analysis of constrained static problems.

Chapter 10 reviews the classical formulations for constrained dynamical systems. D'Alembert's principle, Hamilton's principle, and Lagrange's formulation are updated to accommodate the presence of both holonomic and nonholonomic constraints. The kinematic constraints associated with the lower pair joints are described in details.

The advanced formulations presented in chapter 11 form the theoretical basis for the practical approaches to numerical solutions of multibody systems. Maggi's, the index-1, the null space, and Udwadia and Kalaba's formulations are presented and the chapter concludes with the geometric interpretation of constraints and Gauss' principle.

Finally, chapter 12 describes in a cursory manner the many numerical approaches used to treat constrained dynamical systems, most of which are rooted in the formulations presented in chapter 11. Chapter 12 is in fact a comprehensive review of the literature on methods of constrained dynamics applied to the solution of multibody systems. It is clearly impossible to treat each approach in detail. Rather, the salient features of each approach are given, and the relationships between them are underlined. The chapter concludes with a detailed description of scaling methods for Lagrange's equations of the first kind.

Part 5 presents a comprehensive overview of the many approaches used to parameterize rotation and motion. The vectorial parameterizations of rotation and motion are given special emphasis as they provide a unified approach to this complex topic. Specific parameterizations widely used in multibody formulations are reviewed in details, whereas other are presented in a more cursory manner.

The last part of the book focuses on flexible multibody dynamics problems, which are categorized into three groups: rigid multibody systems, linearly elastic multibody systems, and nonlinearly elastic multibody systems. The last three chapters of the book focus on the latter category, nonlinearly elastic multibody systems. Chapter 15 presents background material. The basic equations of linear elastodynamics are presented first. Next, finite displacement kinematics are studied, with special emphasis on small strain problems.

Chapter 16 develops the governing equations of flexible joints, cables, beams, and plates and shells. All formulations are *geometrically exact*, i.e., all structural components are allowed to undergo arbitrarily large displacements and rotations, although strains are assumed to remain small. Finally, chapter 17 presents details of the implementation of these elements within the framework of finite element formulations. For instance, interpolation of the rotation fields is an issue that requires special attention.

The topics covered in the first three parts of the book form the basis for a three-credit hour, graduate level course in advanced dynamics, typically taken by first year graduate students. Topics selected from the last three parts provide an ample material for a follow-on, three-credit hour, graduate level course in flexible multibody dynamics. The advanced dynamics course is, of course, a prerequisite for the flexible multibody dynamics course.

Typically, engineering students generally grasp concepts more quickly when presented first with practical examples, which then lead to broader generalizations. Consequently, most concepts are first introduced by means of simple examples; more formal and abstract statements are presented later, when the student has a better grasp of the significance of the concepts.

Numerous homework problems are included throughout the book. Some are straightforward applications of basic concepts, others are small projects that require the use of computers and mathematical software, and others involve conceptual questions that are more appropriate for quizzes and exams. The text also provides many examples that treat practical problems in great details. Some of the examples are re-examined in successive chapters to illustrate alternative or more versatile solution methods.

Notation is a challenging issue in dynamics. Given the limitations of the Latin and Greek alphabets, the same symbol is sometimes used to indicate different quantities, but mostly in different contexts. Consequently, no attempt has been made to provide a comprehensive nomenclature, which would lead to even more confusion.

It is traditional to use a bold typeface to represent vectors and tensors, but this is very difficult to reproduce in handwriting, whether on a board or in personal notes. A notation that is more suitable for hand-written notes has been adopted here. Vectors and arrays are denoted using an underline, such as \underline{u} or \underline{F} . Unit vectors are used

frequently and are assigned a special notation using a single overbar, such as \bar{v}_1 , which denotes the first Cartesian coordinate axis. The overbar notation also indicates non-dimensional scalar quantities, *i.e.*, \bar{k} is a non-dimensional stiffness coefficient. This is inconsistent, but the two uses are in such different contexts that it should not lead to confusion. Second-order tensors and matrices are indicated using a double-underline, *i.e.*, $\underline{\underline{R}}$ indicates a 3×3 rotation tensor or $\underline{\underline{M}}$ a $n \times n$ mass matrix.

Notations $\underline{a}^T \underline{b}$, $\tilde{a} \underline{b}$, and $\underline{a} \underline{b}^T$ indicate the scalar, vector, and tensor products, respectively, of two vectors, \underline{a} and \underline{b} . While many students voice their displeasure with this mnemonic convention that departs from the classical “dot” and “cross product” notations, they very rapidly recognize and appreciate its power and conciseness.

Finally, I am indebted to the many students at Georgia Tech who have given me helpful and constructive feedback over the past decade as I developed the course notes that are the precursors of this book. The constructive use of their many questions and confusion has helped shape this book, and the treatment of many topics was modified numerous times before finding their final form.

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