

Contents

Preface	xiii
About the Authors	xv
1 Introduction	1
1.1 Probability Metrics	1
1.2 Applications in Finance	2
2 Probability Distances and Metrics	7
2.1 Introduction	9
2.2 Some Examples of Probability Metrics	9
2.2.1 Engineer's metric	10
2.2.2 Uniform (or Kolmogorov) metric	10
2.2.3 Lévy metric	11
2.2.4 Kantorovich metric	14
2.2.5 L_p -metrics between distribution functions	15
2.2.6 Ky Fan metrics	16
2.2.7 L_p -metric	17
2.3 Distance and Semidistance Spaces	19
2.4 Definitions of Probability Distances and Metrics	24
2.5 Summary	28
2.6 Technical Appendix	28
2.6.1 Universally measurable separable metric spaces	29
2.6.2 The equivalence of the notions of p. (semi-)distance on \mathcal{P}_2 and on \mathfrak{X}	35

3	Choice under Uncertainty	40
3.1	Introduction	41
3.2	Expected Utility Theory	44
3.2.1	St Petersburg Paradox	44
3.2.2	The von Neumann–Morgenstern expected utility theory	46
3.2.3	Types of utility functions	48
3.3	Stochastic Dominance	51
3.3.1	First-order stochastic dominance	52
3.3.2	Second-order stochastic dominance	53
3.3.3	Rothschild–Stiglitz stochastic dominance	55
3.3.4	Third-order stochastic dominance	56
3.3.5	Efficient sets and the portfolio choice problem	58
3.3.6	Return versus payoff	59
3.4	Probability Metrics and Stochastic Dominance	63
3.5	Cumulative Prospect Theory	66
3.6	Summary	70
3.7	Technical Appendix	70
3.7.1	The axioms of choice	71
3.7.2	Stochastic dominance relations of order n	72
3.7.3	Return versus payoff and stochastic dominance	74
3.7.4	Other stochastic dominance relations	76
4	A Classification of Probability Distances	83
4.1	Introduction	86
4.2	Primary Distances and Primary Metrics	86
4.3	Simple Distances and Metrics	90
4.4	Compound Distances and Moment Functions	99
4.5	Ideal Probability Metrics	105
4.5.1	Interpretation and examples of ideal probability metrics	107
4.5.2	Conditions for boundedness of ideal probability metrics	112
4.6	Summary	114
4.7	Technical Appendix	114
4.7.1	Examples of primary distances	114
4.7.2	Examples of simple distances	118
4.7.3	Examples of compound distances	131
4.7.4	Examples of moment functions	135

5	Risk and Uncertainty	146
5.1	Introduction	147
5.2	Measures of Dispersion	150
5.2.1	Standard deviation	151
5.2.2	Mean absolute deviation	153
5.2.3	Semi-standard deviation	154
5.2.4	Axiomatic description	155
5.2.5	Deviation measures	156
5.3	Probability Metrics and Dispersion Measures	158
5.4	Measures of Risk	159
5.4.1	Value-at-risk	160
5.4.2	Computing portfolio VaR in practice	165
5.4.3	Back-testing of VaR	172
5.4.4	Coherent risk measures	175
5.5	Risk Measures and Dispersion Measures	179
5.6	Risk Measures and Stochastic Orders	181
5.7	Summary	182
5.8	Technical Appendix	183
5.8.1	Convex risk measures	183
5.8.2	Probability metrics and deviation measures	184
5.8.3	Deviation measures and probability quasi-metrics	187
6	Average Value-at-Risk	191
6.1	Introduction	192
6.2	Average Value-at-Risk	193
6.2.1	AVaR for stable distributions	200
6.3	AVaR Estimation from a Sample	204
6.4	Computing Portfolio AVaR in Practice	207
6.4.1	The multivariate normal assumption	207
6.4.2	The historical method	208
6.4.3	The hybrid method	208
6.4.4	The Monte Carlo method	209
6.4.5	Kernel methods	211
6.5	Back-testing of AVaR	218
6.6	Spectral Risk Measures	220
6.7	Risk Measures and Probability Metrics	223
6.8	Risk Measures Based on Distortion Functionals	226
6.9	Summary	227

6.10	Technical Appendix	228
6.10.1	Characteristics of conditional loss distributions	228
6.10.2	Higher-order AVaR	232
6.10.3	The minimization formula for AVaR	234
6.10.4	ETL vs AVaR	237
6.10.5	Kernel-based estimation of AVaR	242
6.10.6	Remarks on spectral risk measures	245
7	Computing AVaR through Monte Carlo	252
7.1	Introduction	253
7.2	An Illustration of Monte Carlo Variability	256
7.3	Asymptotic Distribution, Classical Conditions	259
7.4	Rate of Convergence to the Normal Distribution	262
7.4.1	The effect of tail thickness	263
7.4.2	The effect of tail truncation	268
7.4.3	Infinite variance distributions	271
7.5	Asymptotic Distribution, Heavy-tailed Returns	277
7.6	Rate of Convergence, Heavy-tailed Returns	283
7.6.1	Stable Paretian distributions	283
7.6.2	Student's t distribution	286
7.7	On the Choice of a Distributional Model	290
7.7.1	Tail behavior and return frequency	290
7.7.2	Practical implications	295
7.8	Summary	297
7.9	Technical Appendix	298
7.9.1	Proof of the stable limit result	298
8	Stochastic Dominance Revisited	304
8.1	Introduction	306
8.2	Metritzation of Preference Relations	308
8.3	The Hausdorff Metric Structure	310
8.4	Examples	314
8.4.1	The Lévy quasi-semidistance and first-order stochastic dominance	315
8.4.2	Higher-order stochastic dominance	317
8.4.3	The H-quasi-semidistance	320
8.4.4	AVaR generated stochastic orders	322
8.4.5	Compound quasi-semidistances	324
8.5	Utility-type Representations	325

8.6	Almost Stochastic Orders and Degree of Violation	328
8.7	Summary	330
8.8	Technical Appendix	332
8.8.1	Preference relations and topology	332
8.8.2	Quasi-semidistances and preference relations	334
8.8.3	Construction of quasi-semidistances on classes of investors	335
8.8.4	Investors with balanced views	338
8.8.5	Structural classification of probability distances	339
	Index	357