

Contents

1	Introduction	1
1.1	Random Fields on Stratified Manifolds	1
1.2	Topological Complexity	3
1.3	Random Fields and Complexity	7
1.3.1	Statistical Implications	10
1.3.2	Connections with Sample Path Behaviour	10
1.3.3	Geometry	11
2	Gaussian Processes	13
2.1	The Cosine Process	14
2.2	The Cosine Field	16
2.3	Constructing Gaussian Processes	18
2.4	The Canonical Process on $S(\mathbb{R}^l)$	20
2.4.1	The Canonical Processes and Exceedence Probabilities	22
2.4.2	The Canonical Process and Geometry	24
2.5	The Basic Theory of Gaussian Fields	25
2.5.1	Regularity for Gaussian Process	26
2.5.2	Gaussian Fields on \mathbb{R}^N	28
2.5.3	Differentiability	29
2.6	Stationarity, Isotropy, and Constant Variance	31
2.6.1	Spectral Moments and Derivatives of Random Fields	31
2.6.2	Local Isotropy and the Induced Metric	33
2.7	Three Gaussian Facts	34
3	Some Geometry and Some Topology	37
3.1	Some Notation for Riemannian Manifolds	37
3.2	Coarea Formula	39
3.3	Stratified Manifolds	40
3.4	Tube Formulae and Lipschitz-Killing Curvatures	42
3.4.1	Describing Tubes	43
3.4.2	Computing Volumes	46
3.4.3	Intrinsic Volumes	50
3.5	Probabilities of Tubes: Gaussian Minkowski Functionals	51
3.6	Kinematic Formulae	54

3.7	Crofton's Formula	55
3.8	Morse's Theorem	56
4	The Gaussian Kinematic Formula	59
4.1	The Kac-Rice Metatheorem	60
4.2	Real Isotropic Fields on Rectangles: Euler Characteristic	65
4.3	Real Isotropic Fields: Lipschitz-Killing Curvatures	70
4.4	Real Stationary Fields on Rectangles: Euler Characteristic	71
4.5	The Induced Metric and the Need for Riemannian Geometry	73
4.6	The Canonical Isotropic Process on the Sphere	77
4.6.1	A Model Process	77
4.6.2	The GKF for the Model Process	79
4.6.3	Back to the Canonical Process	81
4.7	Fields with Finite Expansions	82
4.8	The GKF in the General Case	83
4.9	Not Just Excursion Sets	84
4.10	Infinite Dimensions	84
5	On Applications: Topological Inference	87
5.1	Local Structure of Extrema and the Euler Characteristic Heuristic	88
5.2	Gaussian Related Random Fields	91
5.2.1	χ^2 Fields	93
5.3	Brain Imaging	94
5.4	Estimating Lipschitz-Killing Curvatures	98
5.5	Cosmology	102
6	Algebraic Topology of Excursion Sets: A New Challenge	107
6.1	Persistent Homology and Barcodes	108
6.1.1	Barcodes of Excursion Sets	108
6.2	Barcode Distributions	110
6.3	The Mean Euler Characteristic of the Barcodes of Gaussian Excursion Sets	112
References	115	
Notation Index	118	
Subject Index	121	