

BRIEF CONTENTS



| | | | |
|-----------------|---|-----|-----|
| | Examples and Applications | 25 | |
| | Preface | 33 | |
| Part I | The Linear Regression Model | | |
| Chapter 1 | Econometrics | 41 | |
| Chapter 2 | The Linear Regression Model | 51 | |
| Chapter 3 | Least Squares | 66 | |
| Chapter 4 | The Least Squares Estimator | 91 | |
| Chapter 5 | Hypothesis Tests and Model Selection | 148 | |
| Chapter 6 | Functional Form and Structural Change | 189 | |
| Chapter 7 | Nonlinear, Semiparametric, and Nonparametric Regression Models | 221 | |
| Chapter 8 | Endogeneity and Instrumental Variable Estimation | 259 | |
| Part II | Generalized Regression Model and Equation Systems | | |
| Chapter 9 | The Generalized Regression Model and Heteroscedasticity | | 297 |
| Chapter 10 | Systems of Equations | 330 | |
| Chapter 11 | Models for Panel Data | 383 | |
| Part III | Estimation Methodology | | |
| Chapter 12 | Estimation Frameworks in Econometrics | 472 | |
| Chapter 13 | Minimum Distance Estimation and the Generalized Method of Moments | 495 | |
| Chapter 14 | Maximum Likelihood Estimation | 549 | |
| Chapter 15 | Simulation-Based Estimation and Inference and Random Parameter Models | 643 | |
| Chapter 16 | Bayesian Estimation and Inference | 695 | |
| Part IV | Cross Sections, Panel Data, and Microeconometrics | | |
| Chapter 17 | Discrete Choice | 721 | |
| Chapter 18 | Discrete Choices and Event Counts | 800 | |
| Chapter 19 | Limited Dependent Variables—Truncation, Censoring, and Sample Selection | 873 | |

Part V Time Series and Macroeconometrics

| | | |
|------------|--------------------|-----|
| Chapter 20 | Serial Correlation | 943 |
| Chapter 21 | Nonstationary Data | 982 |

Part VI Appendices

| | | |
|-----------------------------------|-------------------------------------|------|
| Appendix A | Matrix Algebra | 1013 |
| Appendix B | Probability and Distribution Theory | 1055 |
| Appendix C | Estimation and Inference | 1087 |
| Appendix D | Large-Sample Distribution Theory | 1106 |
| Appendix E | Computation and Optimization | 1129 |
| Appendix F | Data Sets Used in Applications | 1149 |
| References | | 1155 |
| Combined Author and Subject Index | | 1201 |

CONTENTS



Examples and Applications 25

Preface 33

PART I The Linear Regression Model

CHAPTER 1 Econometrics 41

- 1.1 Introduction 41
- 1.2 The Paradigm of Econometrics 41
- 1.3 The Practice of Econometrics 43
- 1.4 Econometric Modeling 44
- 1.5 Plan of the Book 47
- 1.6 Preliminaries 49
 - 1.6.1 *Numerical Examples* 49
 - 1.6.2 *Software and Replication* 49
 - 1.6.3 *Notational Conventions* 49

CHAPTER 2 The Linear Regression Model 51

- 2.1 Introduction 51
- 2.2 The Linear Regression Model 52
- 2.3 Assumptions of the Linear Regression Model 55
 - 2.3.1 *Linearity of the Regression Model* 55
 - 2.3.2 *Full Rank* 59
 - 2.3.3 *Regression* 60
 - 2.3.4 *Spherical Disturbances* 61
 - 2.3.5 *Data Generating Process for the Regressors* 63
 - 2.3.6 *Normality* 63
 - 2.3.7 *Independence* 64
- 2.4 Summary and Conclusions 65

CHAPTER 3 Least Squares 66

- 3.1 Introduction 66
- 3.2 Least Squares Regression 66
 - 3.2.1 *The Least Squares Coefficient Vector* 67

| | | |
|------------------|---|-----------|
| 3.2.2 | <i>Application: An Investment Equation</i> | 68 |
| 3.2.3 | <i>Algebraic Aspects of the Least Squares Solution</i> | 70 |
| 3.2.4 | <i>Projection</i> | 71 |
| 3.3 | Partitioned Regression and Partial Regression | 72 |
| 3.4 | Partial Regression and Partial Correlation Coefficients | 76 |
| 3.5 | Goodness of Fit and the Analysis of Variance | 79 |
| 3.5.1 | <i>The Adjusted R-Squared and a Measure of Fit</i> | 82 |
| 3.5.2 | <i>R-Squared and the Constant Term in the Model</i> | 84 |
| 3.5.3 | <i>Comparing Models</i> | 85 |
| 3.6 | Linearly Transformed Regression | 86 |
| 3.7 | Summary and Conclusions | 87 |
| | | |
| CHAPTER 4 | The Least Squares Estimator | 91 |
| 4.1 | Introduction | 91 |
| 4.2 | Motivating Least Squares | 92 |
| 4.2.1 | <i>The Population Orthogonality Conditions</i> | 92 |
| 4.2.2 | <i>Minimum Mean Squared Error Predictor</i> | 93 |
| 4.2.3 | <i>Minimum Variance Linear Unbiased Estimation</i> | 94 |
| 4.3 | Finite Sample Properties of Least Squares | 94 |
| 4.3.1 | <i>Unbiased Estimation</i> | 95 |
| 4.3.2 | <i>Bias Caused by Omission of Relevant Variables</i> | 96 |
| 4.3.3 | <i>Inclusion of Irrelevant Variables</i> | 98 |
| 4.3.4 | <i>The Variance of the Least Squares Estimator</i> | 98 |
| 4.3.5 | <i>The Gauss–Markov Theorem</i> | 100 |
| 4.3.6 | <i>The Implications of Stochastic Regressors</i> | 100 |
| 4.3.7 | <i>Estimating the Variance of the Least Squares Estimator</i> | 101 |
| 4.3.8 | <i>The Normality Assumption</i> | 103 |
| 4.4 | Large Sample Properties of the Least Squares Estimator | 103 |
| 4.4.1 | <i>Consistency of the Least Squares Estimator of β</i> | 103 |
| 4.4.2 | <i>Asymptotic Normality of the Least Squares Estimator</i> | 105 |
| 4.4.3 | <i>Consistency of s^2 and the Estimator of Asy. Var[\mathbf{b}]</i> | 107 |
| 4.4.4 | <i>Asymptotic Distribution of a Function of \mathbf{b}: The Delta Method</i> | 108 |
| 4.4.5 | <i>Asymptotic Efficiency</i> | 109 |
| 4.4.6 | <i>Maximum Likelihood Estimation</i> | 113 |
| 4.5 | Interval Estimation | 115 |
| 4.5.1 | <i>Forming a Confidence Interval for a Coefficient</i> | 116 |
| 4.5.2 | <i>Confidence Intervals Based on Large Samples</i> | 118 |
| 4.5.3 | <i>Confidence Interval for a Linear Combination of Coefficients: The Oaxaca Decomposition</i> | 119 |
| 4.6 | Prediction and Forecasting | 120 |
| 4.6.1 | <i>Prediction Intervals</i> | 121 |
| 4.6.2 | <i>Predicting y When the Regression Model Describes $\log y$</i> | 121 |

| | | |
|---|--|------------|
| 4.6.3 | <i>Prediction Interval for y When the Regression Model Describes Log y</i> | 123 |
| 4.6.4 | <i>Forecasting</i> | 127 |
| 4.7 | Data Problems | 128 |
| 4.7.1 | <i>Multicollinearity</i> | 129 |
| 4.7.2 | <i>Pretest Estimation</i> | 131 |
| 4.7.3 | <i>Principal Components</i> | 132 |
| 4.7.4 | <i>Missing Values and Data Imputation</i> | 134 |
| 4.7.5 | <i>Measurement Error</i> | 137 |
| 4.7.6 | <i>Outliers and Influential Observations</i> | 139 |
| 4.8 | Summary and Conclusions | 142 |
| CHAPTER 5 Hypothesis Tests and Model Selection | | 148 |
| 5.1 | Introduction | 148 |
| 5.2 | Hypothesis Testing Methodology | 148 |
| 5.2.1 | <i>Restrictions and Hypotheses</i> | 149 |
| 5.2.2 | <i>Nested Models</i> | 150 |
| 5.2.3 | <i>Testing Procedures—Neyman–Pearson Methodology</i> | 151 |
| 5.2.4 | <i>Size, Power, and Consistency of a Test</i> | 151 |
| 5.2.5 | <i>A Methodological Dilemma: Bayesian versus Classical Testing</i> | 152 |
| 5.3 | Two Approaches to Testing Hypotheses | 152 |
| 5.4 | Wald Tests Based on the Distance Measure | 155 |
| 5.4.1 | <i>Testing a Hypothesis about a Coefficient</i> | 155 |
| 5.4.2 | <i>The F Statistic and the Least Squares Discrepancy</i> | 157 |
| 5.5 | Testing Restrictions Using the Fit of the Regression | 161 |
| 5.5.1 | <i>The Restricted Least Squares Estimator</i> | 161 |
| 5.5.2 | <i>The Loss of Fit from Restricted Least Squares</i> | 162 |
| 5.5.3 | <i>Testing the Significance of the Regression</i> | 166 |
| 5.5.4 | <i>Solving Out the Restrictions and a Caution about Using R^2</i> | 166 |
| 5.6 | Nonnormal Disturbances and Large-Sample Tests | 167 |
| 5.7 | Testing Nonlinear Restrictions | 171 |
| 5.8 | Choosing between Nonnested Models | 174 |
| 5.8.1 | <i>Testing Nonnested Hypotheses</i> | 174 |
| 5.8.2 | <i>An Encompassing Model</i> | 175 |
| 5.8.3 | <i>Comprehensive Approach—The J Test</i> | 176 |
| 5.9 | A Specification Test | 177 |
| 5.10 | Model Building—A General to Simple Strategy | 178 |
| 5.10.1 | <i>Model Selection Criteria</i> | 179 |
| 5.10.2 | <i>Model Selection</i> | 180 |
| 5.10.3 | <i>Classical Model Selection</i> | 180 |
| 5.10.4 | <i>Bayesian Model Averaging</i> | 181 |
| 5.11 | Summary and Conclusions | 183 |

| | | |
|------------------|---|------------|
| CHAPTER 6 | Functional Form and Structural Change | 189 |
| 6.1 | Introduction | 189 |
| 6.2 | Using Binary Variables | 189 |
| 6.2.1 | <i>Binary Variables in Regression</i> | 189 |
| 6.2.2 | <i>Several Categories</i> | 192 |
| 6.2.3 | <i>Several Groupings</i> | 192 |
| 6.2.4 | <i>Threshold Effects and Categorical Variables</i> | 194 |
| 6.2.5 | <i>Treatment Effects and Differences in Differences Regression</i> | 195 |
| 6.3 | Nonlinearity in the Variables | 198 |
| 6.3.1 | <i>Piecewise Linear Regression</i> | 198 |
| 6.3.2 | <i>Functional Forms</i> | 200 |
| 6.3.3 | <i>Interaction Effects</i> | 201 |
| 6.3.4 | <i>Identifying Nonlinearity</i> | 202 |
| 6.3.5 | <i>Intrinsically Linear Models</i> | 205 |
| 6.4 | Modeling and Testing for a Structural Break | 208 |
| 6.4.1 | <i>Different Parameter Vectors</i> | 208 |
| 6.4.2 | <i>Insufficient Observations</i> | 209 |
| 6.4.3 | <i>Change in a Subset of Coefficients</i> | 210 |
| 6.4.4 | <i>Tests of Structural Break with Unequal Variances</i> | 211 |
| 6.4.5 | <i>Predictive Test of Model Stability</i> | 214 |
| 6.5 | Summary and Conclusions | 215 |
| CHAPTER 7 | Nonlinear, Semiparametric, and Nonparametric Regression Models | 221 |
| 7.1 | Introduction | 221 |
| 7.2 | Nonlinear Regression Models | 222 |
| 7.2.1 | <i>Assumptions of the Nonlinear Regression Model</i> | 222 |
| 7.2.2 | <i>The Nonlinear Least Squares Estimator</i> | 224 |
| 7.2.3 | <i>Large Sample Properties of the Nonlinear Least Squares Estimator</i> | 226 |
| 7.2.4 | <i>Hypothesis Testing and Parametric Restrictions</i> | 229 |
| 7.2.5 | <i>Applications</i> | 231 |
| 7.2.6 | <i>Computing the Nonlinear Least Squares Estimator</i> | 240 |
| 7.3 | Median and Quantile Regression | 242 |
| 7.3.1 | <i>Least Absolute Deviations Estimation</i> | 243 |
| 7.3.2 | <i>Quantile Regression Models</i> | 247 |
| 7.4 | Partially Linear Regression | 250 |
| 7.5 | Nonparametric Regression | 252 |
| 7.6 | Summary and Conclusions | 255 |
| CHAPTER 8 | Endogeneity and Instrumental Variable Estimation | 259 |
| 8.1 | Introduction | 259 |
| 8.2 | Assumptions of the Extended Model | 263 |

| | | |
|-------|--|-----|
| 8.3 | Estimation | 264 |
| 8.3.1 | <i>Least Squares</i> | 265 |
| 8.3.2 | <i>The Instrumental Variables Estimator</i> | 265 |
| 8.3.3 | <i>Motivating the Instrumental Variables Estimator</i> | 267 |
| 8.3.4 | <i>Two-Stage Least Squares</i> | 270 |
| 8.4 | Two Specification Tests | 273 |
| 8.4.1 | <i>The Hausman and Wu Specification Tests</i> | 274 |
| 8.4.2 | <i>A Test for Overidentification</i> | 278 |
| 8.5 | Measurement Error | 279 |
| 8.5.1 | <i>Least Squares Attenuation</i> | 280 |
| 8.5.2 | <i>Instrumental Variables Estimation</i> | 282 |
| 8.5.3 | <i>Proxy Variables</i> | 282 |
| 8.6 | Nonlinear Instrumental Variables Estimation | 286 |
| 8.7 | Weak Instruments | 289 |
| 8.8 | Natural Experiments and the Search for Causal Effects | 291 |
| 8.9 | Summary and Conclusions | 294 |

PART II Generalized Regression Model and Equation Systems

CHAPTER 9 The Generalized Regression Model and Heteroscedasticity 297

| | | |
|-------|--|-----|
| 9.1 | Introduction | 297 |
| 9.2 | Inefficient Estimation by Least Squares and Instrumental Variables | 298 |
| 9.2.1 | <i>Finite-Sample Properties of Ordinary Least Squares</i> | 299 |
| 9.2.2 | <i>Asymptotic Properties of Ordinary Least Squares</i> | 299 |
| 9.2.3 | <i>Robust Estimation of Asymptotic Covariance Matrices</i> | 301 |
| 9.2.4 | <i>Instrumental Variable Estimation</i> | 302 |
| 9.3 | Efficient Estimation by Generalized Least Squares | 304 |
| 9.3.1 | <i>Generalized Least Squares (GLS)</i> | 304 |
| 9.3.2 | <i>Feasible Generalized Least Squares (FGLS)</i> | 306 |
| 9.4 | Heteroscedasticity and Weighted Least Squares | 308 |
| 9.4.1 | <i>Ordinary Least Squares Estimation</i> | 309 |
| 9.4.2 | <i>Inefficiency of Ordinary Least Squares</i> | 310 |
| 9.4.3 | <i>The Estimated Covariance Matrix of \mathbf{b}</i> | 310 |
| 9.4.4 | <i>Estimating the Appropriate Covariance Matrix for Ordinary Least Squares</i> | 312 |
| 9.5 | Testing for Heteroscedasticity | 315 |
| 9.5.1 | <i>White's General Test</i> | 315 |
| 9.5.2 | <i>The Breusch-Pagan/Godfrey LM Test</i> | 316 |
| 9.6 | Weighted Least Squares | 317 |
| 9.6.1 | <i>Weighted Least Squares with Known Ω</i> | 318 |
| 9.6.2 | <i>Estimation When Ω Contains Unknown Parameters</i> | 319 |

- 9.7 Applications 320
 - 9.7.1 *Multiplicative Heteroscedasticity* 320
 - 9.7.2 *Groupwise Heteroscedasticity* 322
- 9.8 Summary and Conclusions 325

CHAPTER 10 Systems of Equations 330

- 10.1 Introduction 330
- 10.2 The Seemingly Unrelated Regressions Model 332
 - 10.2.1 *Generalized Least Squares* 333
 - 10.2.2 *Seemingly Unrelated Regressions with Identical Regressors* 335
 - 10.2.3 *Feasible Generalized Least Squares* 336
 - 10.2.4 *Testing Hypotheses* 336
 - 10.2.5 *A Specification Test for the SUR Model* 337
 - 10.2.6 *The Pooled Model* 339
- 10.3 Seemingly Unrelated Generalized Regression Models 344
- 10.4 Nonlinear Systems of Equations 345
- 10.5 Systems of Demand Equations: Singular Systems 347
 - 10.5.1 *Cobb–Douglas Cost Function* 347
 - 10.5.2 *Flexible Functional Forms: The Translog Cost Function* 350
- 10.6 Simultaneous Equations Models 354
 - 10.6.1 *Systems of Equations* 355
 - 10.6.2 *A General Notation for Linear Simultaneous Equations Models* 358
 - 10.6.3 *The Problem of Identification* 361
 - 10.6.4 *Single Equation Estimation and Inference* 366
 - 10.6.5 *System Methods of Estimation* 369
 - 10.6.6 *Testing in the Presence of Weak Instruments* 374
- 10.7 Summary and Conclusions 376

CHAPTER 11 Models for Panel Data 383

- 11.1 Introduction 383
- 11.2 Panel Data Models 384
 - 11.2.1 *General Modeling Framework for Analyzing Panel Data* 385
 - 11.2.2 *Model Structures* 386
 - 11.2.3 *Extensions* 387
 - 11.2.4 *Balanced and Unbalanced Panels* 388
 - 11.2.5 *Well-Behaved Panel Data* 388
- 11.3 The Pooled Regression Model 389
 - 11.3.1 *Least Squares Estimation of the Pooled Model* 389
 - 11.3.2 *Robust Covariance Matrix Estimation* 390
 - 11.3.3 *Clustering and Stratification* 392
 - 11.3.4 *Robust Estimation Using Group Means* 394

| | | |
|---------|---|-----|
| 11.3.5 | <i>Estimation with First Differences</i> | 395 |
| 11.3.6 | <i>The Within- and Between-Groups Estimators</i> | 397 |
| 11.4 | The Fixed Effects Model | 399 |
| 11.4.1 | <i>Least Squares Estimation</i> | 400 |
| 11.4.2 | <i>Small T Asymptotics</i> | 402 |
| 11.4.3 | <i>Testing the Significance of the Group Effects</i> | 403 |
| 11.4.4 | <i>Fixed Time and Group Effects</i> | 403 |
| 11.4.5 | <i>Time-Invariant Variables and Fixed Effects Vector Decomposition</i> | 404 |
| 11.5 | Random Effects | 410 |
| 11.5.1 | <i>Least Squares Estimation</i> | 412 |
| 11.5.2 | <i>Generalized Least Squares</i> | 413 |
| 11.5.3 | <i>Feasible Generalized Least Squares When Σ Is Unknown</i> | 414 |
| 11.5.4 | <i>Testing for Random Effects</i> | 416 |
| 11.5.5 | <i>Hausman's Specification Test for the Random Effects Model</i> | 419 |
| 11.5.6 | <i>Extending the Unobserved Effects Model: Mundlak's Approach</i> | 420 |
| 11.5.7 | <i>Extending the Random and Fixed Effects Models: Chamberlain's Approach</i> | 421 |
| 11.6 | Nonspherical Disturbances and Robust Covariance Estimation | 425 |
| 11.6.1 | <i>Robust Estimation of the Fixed Effects Model</i> | 425 |
| 11.6.2 | <i>Heteroscedasticity in the Random Effects Model</i> | 427 |
| 11.6.3 | <i>Autocorrelation in Panel Data Models</i> | 428 |
| 11.6.4 | <i>Cluster (and Panel) Robust Covariance Matrices for Fixed and Random Effects Estimators</i> | 428 |
| 11.7 | Spatial Autocorrelation | 429 |
| 11.8 | Endogeneity | 434 |
| 11.8.1 | <i>Hausman and Taylor's Instrumental Variables Estimator</i> | 434 |
| 11.8.2 | <i>Consistent Estimation of Dynamic Panel Data Models: Anderson and Hsiao's IV Estimator</i> | 438 |
| 11.8.3 | <i>Efficient Estimation of Dynamic Panel Data Models—The Arellano/Bond Estimators</i> | 440 |
| 11.8.4 | <i>Nonstationary Data and Panel Data Models</i> | 450 |
| 11.9 | Nonlinear Regression with Panel Data | 451 |
| 11.9.1 | <i>A Robust Covariance Matrix for Nonlinear Least Squares</i> | 451 |
| 11.9.2 | <i>Fixed Effects</i> | 452 |
| 11.9.3 | <i>Random Effects</i> | 454 |
| 11.10 | Systems of Equations | 455 |
| 11.11 | Parameter Heterogeneity | 456 |
| 11.11.1 | <i>The Random Coefficients Model</i> | 457 |
| 11.11.2 | <i>A Hierarchical Linear Model</i> | 460 |
| 11.11.3 | <i>Parameter Heterogeneity and Dynamic Panel Data Models</i> | 461 |
| 11.12 | Summary and Conclusions | 466 |

PART III Estimation Methodology

| | |
|---|------------|
| CHAPTER 12 Estimation Frameworks in Econometrics | 472 |
| 12.1 Introduction | 472 |
| 12.2 Parametric Estimation and Inference | 474 |
| 12.2.1 <i>Classical Likelihood-Based Estimation</i> | 474 |
| 12.2.2 <i>Modeling Joint Distributions with Copula Functions</i> | 476 |
| 12.3 Semiparametric Estimation | 479 |
| 12.3.1 <i>GMM Estimation in Econometrics</i> | 479 |
| 12.3.2 <i>Maximum Empirical Likelihood Estimation</i> | 480 |
| 12.3.3 <i>Least Absolute Deviations Estimation and Quantile Regression</i> | 481 |
| 12.3.4 <i>Kernel Density Methods</i> | 482 |
| 12.3.5 <i>Comparing Parametric and Semiparametric Analyses</i> | 483 |
| 12.4 Nonparametric Estimation | 484 |
| 12.4.1 <i>Kernel Density Estimation</i> | 485 |
| 12.5 Properties of Estimators | 487 |
| 12.5.1 <i>Statistical Properties of Estimators</i> | 488 |
| 12.5.2 <i>Extremum Estimators</i> | 489 |
| 12.5.3 <i>Assumptions for Asymptotic Properties of Extremum Estimators</i> | 489 |
| 12.5.4 <i>Asymptotic Properties of Estimators</i> | 492 |
| 12.5.5 <i>Testing Hypotheses</i> | 493 |
| 12.6 Summary and Conclusions | 494 |
| | |
| CHAPTER 13 Minimum Distance Estimation and the Generalized Method of Moments | 495 |
| 13.1 Introduction | 495 |
| 13.2 Consistent Estimation: The Method of Moments | 496 |
| 13.2.1 <i>Random Sampling and Estimating the Parameters of Distributions</i> | 497 |
| 13.2.2 <i>Asymptotic Properties of the Method of Moments Estimator</i> | 501 |
| 13.2.3 <i>Summary—The Method of Moments</i> | 503 |
| 13.3 Minimum Distance Estimation | 503 |
| 13.4 The Generalized Method of Moments (GMM) Estimator | 508 |
| 13.4.1 <i>Estimation Based on Orthogonality Conditions</i> | 508 |
| 13.4.2 <i>Generalizing the Method of Moments</i> | 510 |
| 13.4.3 <i>Properties of the GMM Estimator</i> | 514 |
| 13.5 Testing Hypotheses in the GMM Framework | 519 |
| 13.5.1 <i>Testing the Validity of the Moment Restrictions</i> | 519 |
| 13.5.2 <i>GMM Counterparts to the WALD, LM, and LR Tests</i> | 520 |

14 Contents

| | | |
|--------|--|-----|
| 13.6 | GMM Estimation of Econometric Models | 522 |
| 13.6.1 | <i>Single-Equation Linear Models</i> | 522 |
| 13.6.2 | <i>Single-Equation Nonlinear Models</i> | 528 |
| 13.6.3 | <i>Seemingly Unrelated Regression Models</i> | 531 |
| 13.6.4 | <i>Simultaneous Equations Models with Heteroscedasticity</i> | 533 |
| 13.6.5 | <i>GMM Estimation of Dynamic Panel Data Models</i> | 536 |
| 13.7 | Summary and Conclusions | 547 |

CHAPTER 14 Maximum Likelihood Estimation 549

| | | |
|----------|--|-----|
| 14.1 | Introduction | 549 |
| 14.2 | The Likelihood Function and Identification of the Parameters | 549 |
| 14.3 | Efficient Estimation: The Principle of Maximum Likelihood | 551 |
| 14.4 | Properties of Maximum Likelihood Estimators | 553 |
| 14.4.1 | <i>Regularity Conditions</i> | 554 |
| 14.4.2 | <i>Properties of Regular Densities</i> | 555 |
| 14.4.3 | <i>The Likelihood Equation</i> | 557 |
| 14.4.4 | <i>The Information Matrix Equality</i> | 557 |
| 14.4.5 | <i>Asymptotic Properties of the Maximum Likelihood Estimator</i> | 557 |
| 14.4.5.a | <i>Consistency</i> | 558 |
| 14.4.5.b | <i>Asymptotic Normality</i> | 559 |
| 14.4.5.c | <i>Asymptotic Efficiency</i> | 560 |
| 14.4.5.d | <i>Invariance</i> | 561 |
| 14.4.5.e | <i>Conclusion</i> | 561 |
| 14.4.6 | <i>Estimating the Asymptotic Variance of the Maximum Likelihood Estimator</i> | 561 |
| 14.5 | Conditional Likelihoods, Econometric Models, and the GMM Estimator | 563 |
| 14.6 | Hypothesis and Specification Tests and Fit Measures | 564 |
| 14.6.1 | <i>The Likelihood Ratio Test</i> | 566 |
| 14.6.2 | <i>The Wald Test</i> | 567 |
| 14.6.3 | <i>The Lagrange Multiplier Test</i> | 569 |
| 14.6.4 | <i>An Application of the Likelihood-Based Test Procedures</i> | 571 |
| 14.6.5 | <i>Comparing Models and Computing Model Fit</i> | 573 |
| 14.6.6 | <i>Vuong's Test and the Kullback–Leibler Information Criterion</i> | 574 |
| 14.7 | Two-Step Maximum Likelihood Estimation | 576 |
| 14.8 | Pseudo-Maximum Likelihood Estimation and Robust Asymptotic Covariance Matrices | 582 |
| 14.8.1 | <i>Maximum Likelihood and GMM Estimation</i> | 583 |
| 14.8.2 | <i>Maximum Likelihood and M Estimation</i> | 583 |
| 14.8.3 | <i>Sandwich Estimators</i> | 585 |
| 14.8.4 | <i>Cluster Estimators</i> | 586 |

| | | |
|----------|---|-----|
| 14.9 | Applications of Maximum Likelihood Estimation | 588 |
| 14.9.1 | <i>The Normal Linear Regression Model</i> | 588 |
| 14.9.2 | <i>The Generalized Regression Model</i> | 592 |
| 14.9.2.a | <i>Multiplicative Heteroscedasticity</i> | 594 |
| 14.9.2.b | <i>Autocorrelation</i> | 597 |
| 14.9.3 | <i>Seemingly Unrelated Regression Models</i> | 600 |
| 14.9.3.a | <i>The Pooled Model</i> | 600 |
| 14.9.3.b | <i>The SUR Model</i> | 602 |
| 14.9.3.c | <i>Exclusion Restrictions</i> | 602 |
| 14.9.4 | <i>Simultaneous Equations Models</i> | 607 |
| 14.9.5 | <i>Maximum Likelihood Estimation of Nonlinear Regression Models</i> | 608 |
| 14.9.6 | <i>Panel Data Applications</i> | 613 |
| 14.9.6.a | <i>ML Estimation of the Linear Random Effects Model</i> | 614 |
| 14.9.6.b | <i>Nested Random Effects</i> | 616 |
| 14.9.6.c | <i>Random Effects in Nonlinear Models: MLE Using Quadrature</i> | 620 |
| 14.9.6.d | <i>Fixed Effects in Nonlinear Models: Full MLE</i> | 624 |
| 14.10 | Latent Class and Finite Mixture Models | 628 |
| 14.10.1 | <i>A Finite Mixture Model</i> | 629 |
| 14.10.2 | <i>Measured and Unmeasured Heterogeneity</i> | 631 |
| 14.10.3 | <i>Predicting Class Membership</i> | 631 |
| 14.10.4 | <i>A Conditional Latent Class Model</i> | 632 |
| 14.10.5 | <i>Determining the Number of Classes</i> | 634 |
| 14.10.6 | <i>A Panel Data Application</i> | 635 |
| 14.11 | Summary and Conclusions | 638 |

CHAPTER 15 Simulation-Based Estimation and Inference and Random Parameter Models 643

| | | |
|--------|--|-----|
| 15.1 | Introduction | 643 |
| 15.2 | Random Number Generation | 645 |
| 15.2.1 | <i>Generating Pseudo-Random Numbers</i> | 645 |
| 15.2.2 | <i>Sampling from a Standard Uniform Population</i> | 646 |
| 15.2.3 | <i>Sampling from Continuous Distributions</i> | 647 |
| 15.2.4 | <i>Sampling from a Multivariate Normal Population</i> | 648 |
| 15.2.5 | <i>Sampling from Discrete Populations</i> | 648 |
| 15.3 | Simulation-Based Statistical Inference: The Method of Krinsky and Robb | 649 |
| 15.4 | Bootstrapping Standard Errors and Confidence Intervals | 651 |
| 15.5 | Monte Carlo Studies | 655 |
| 15.5.1 | <i>A Monte Carlo Study: Behavior of a Test Statistic</i> | 657 |
| 15.5.2 | <i>A Monte Carlo Study: The Incidental Parameters Problem</i> | 659 |
| 15.6 | Simulation-Based Estimation | 661 |
| 15.6.1 | <i>Random Effects in a Nonlinear Model</i> | 661 |

| | | |
|--|--|------------|
| 15.6.2 | <i>Monte Carlo Integration</i> | 663 |
| 15.6.2.a | <i>Halton Sequences and Random Draws for Simulation-Based Integration</i> | 665 |
| 15.6.2.b | <i>Computing Multivariate Normal Probabilities Using the GHK Simulator</i> | 667 |
| 15.6.3 | <i>Simulation-Based Estimation of Random Effects Models</i> | 669 |
| 15.7 | A Random Parameters Linear Regression Model | 674 |
| 15.8 | Hierarchical Linear Models | 679 |
| 15.9 | Nonlinear Random Parameter Models | 681 |
| 15.10 | Individual Parameter Estimates | 682 |
| 15.11 | Mixed Models and Latent Class Models | 690 |
| 15.12 | Summary and Conclusions | 693 |
| | | |
| CHAPTER 16 | Bayesian Estimation and Inference | 695 |
| 16.1 | Introduction | 695 |
| 16.2 | Bayes Theorem and the Posterior Density | 696 |
| 16.3 | Bayesian Analysis of the Classical Regression Model | 698 |
| 16.3.1 | <i>Analysis with a Noninformative Prior</i> | 699 |
| 16.3.2 | <i>Estimation with an Informative Prior Density</i> | 701 |
| 16.4 | Bayesian Inference | 704 |
| 16.4.1 | <i>Point Estimation</i> | 704 |
| 16.4.2 | <i>Interval Estimation</i> | 705 |
| 16.4.3 | <i>Hypothesis Testing</i> | 706 |
| 16.4.4 | <i>Large-Sample Results</i> | 708 |
| 16.5 | Posterior Distributions and the Gibbs Sampler | 708 |
| 16.6 | Application: Binomial Probit Model | 711 |
| 16.7 | Panel Data Application: Individual Effects Models | 714 |
| 16.8 | Hierarchical Bayes Estimation of a Random Parameters Model | 716 |
| 16.9 | Summary and Conclusions | 718 |
| | | |
| PART IV Cross Sections, Panel Data, and Microeconometrics | | |
| CHAPTER 17 | Discrete Choice | 721 |
| 17.1 | Introduction | 721 |
| 17.2 | Models for Binary Outcomes | 723 |
| 17.2.1 | <i>Random Utility Models for Individual Choice</i> | 724 |
| 17.2.2 | <i>A Latent Regression Model</i> | 726 |
| 17.2.3 | <i>Functional Form and Regression</i> | 727 |
| 17.3 | Estimation and Inference in Binary Choice Models | 730 |
| 17.3.1 | <i>Robust Covariance Matrix Estimation</i> | 732 |
| 17.3.2 | <i>Marginal Effects and Average Partial Effects</i> | 733 |

| | | | |
|------|----------|--|-----|
| | 17.3.2.a | Average Partial Effects | 736 |
| | 17.3.2.b | Interaction Effects | 739 |
| | 17.3.3 | Measuring Goodness of Fit | 741 |
| | 17.3.4 | Hypothesis Tests | 743 |
| | 17.3.5 | Endogenous Right-Hand-Side Variables in Binary Choice Models | 746 |
| | 17.3.6 | Endogenous Choice-Based Sampling | 750 |
| | 17.3.7 | Specification Analysis | 751 |
| | 17.3.7.a | Omitted Variables | 753 |
| | 17.3.7.b | Heteroscedasticity | 754 |
| 17.4 | | Binary Choice Models for Panel Data | 756 |
| | 17.4.1 | The Pooled Estimator | 757 |
| | 17.4.2 | Random Effects Models | 758 |
| | 17.4.3 | Fixed Effects Models | 761 |
| | 17.4.4 | A Conditional Fixed Effects Estimator | 762 |
| | 17.4.5 | Mundlak's Approach, Variable Addition, and Bias Reduction | 767 |
| | 17.4.6 | Dynamic Binary Choice Models | 769 |
| | 17.4.7 | A Semiparametric Model for Individual Heterogeneity | 771 |
| | 17.4.8 | Modeling Parameter Heterogeneity | 773 |
| | 17.4.9 | Nonresponse, Attrition, and Inverse Probability Weighting | 774 |
| 17.5 | | Bivariate and Multivariate Probit Models | 778 |
| | 17.5.1 | Maximum Likelihood Estimation | 779 |
| | 17.5.2 | Testing for Zero Correlation | 782 |
| | 17.5.3 | Partial Effects | 782 |
| | 17.5.4 | A Panel Data Model for Bivariate Binary Response | 784 |
| | 17.5.5 | Endogenous Binary Variable in a Recursive Bivariate Probit Model | 785 |
| | 17.5.6 | Endogenous Sampling in a Binary Choice Model | 789 |
| | 17.5.7 | A Multivariate Probit Model | 792 |
| 17.6 | | Summary and Conclusions | 795 |

CHAPTER 18 Discrete Choices and Event Counts 800

| | | | |
|------|--------|--|-----|
| 18.1 | | Introduction | 800 |
| 18.2 | | Models for Unordered Multiple Choices | 801 |
| | 18.2.1 | Random Utility Basis of the Multinomial Logit Model | 801 |
| | 18.2.2 | The Multinomial Logit Model | 803 |
| | 18.2.3 | The Conditional Logit Model | 806 |
| | 18.2.4 | The Independence from Irrelevant Alternatives Assumption | 807 |
| | 18.2.5 | Nested Logit Models | 808 |
| | 18.2.6 | The Multinomial Probit Model | 810 |
| | 18.2.7 | The Mixed Logit Model | 811 |
| | 18.2.8 | A Generalized Mixed Logit Model | 812 |

| | | |
|---------|--|-----|
| 18.2.9 | <i>Application: Conditional Logit Model for Travel Mode Choice</i> | 813 |
| 18.2.10 | <i>Estimating Willingness to Pay</i> | 819 |
| 18.2.11 | <i>Panel Data and Stated Choice Experiments</i> | 821 |
| 18.2.12 | <i>Aggregate Market Share Data—The BLP Random Parameters Model</i> | 822 |
| 18.3 | Random Utility Models for Ordered Choices | 824 |
| 18.3.1 | <i>The Ordered Probit Model</i> | 827 |
| 18.3.2 | <i>A Specification Test for the Ordered Choice Model</i> | 831 |
| 18.3.3 | <i>Bivariate Ordered Probit Models</i> | 832 |
| 18.3.4 | <i>Panel Data Applications</i> | 834 |
| | 18.3.4.a <i>Ordered Probit Models with Fixed Effects</i> | 834 |
| | 18.3.4.b <i>Ordered Probit Models with Random Effects</i> | 835 |
| 18.3.5 | <i>Extensions of the Ordered Probit Model</i> | 838 |
| | 18.3.5.a <i>Threshold Models—Generalized Ordered Choice Models</i> | 839 |
| | 18.3.5.b <i>Thresholds and Heterogeneity—Anchoring Vignettes</i> | 840 |
| 18.4 | Models for Counts of Events | 842 |
| 18.4.1 | <i>The Poisson Regression Model</i> | 843 |
| 18.4.2 | <i>Measuring Goodness of Fit</i> | 844 |
| 18.4.3 | <i>Testing for Overdispersion</i> | 845 |
| 18.4.4 | <i>Heterogeneity and the Negative Binomial Regression Model</i> | 846 |
| 18.4.5 | <i>Functional Forms for Count Data Models</i> | 847 |
| 18.4.6 | <i>Truncation and Censoring in Models for Counts</i> | 850 |
| 18.4.7 | <i>Panel Data Models</i> | 855 |
| | 18.4.7.a <i>Robust Covariance Matrices for Pooled Estimators</i> | 856 |
| | 18.4.7.b <i>Fixed Effects</i> | 857 |
| | 18.4.7.c <i>Random Effects</i> | 858 |
| 18.4.8 | <i>Two-Part Models: Zero Inflation and Hurdle Models</i> | 861 |
| 18.4.9 | <i>Endogenous Variables and Endogenous Participation</i> | 866 |
| 18.5 | Summary and Conclusions | 869 |

CHAPTER 19 Limited Dependent Variables—Truncation, Censoring, and Sample Selection 873

| | | |
|------|--|-----|
| 19.1 | Introduction | 873 |
| 19.2 | Truncation | 873 |
| | 19.2.1 <i>Truncated Distributions</i> | 874 |
| | 19.2.2 <i>Moments of Truncated Distributions</i> | 875 |
| | 19.2.3 <i>The Truncated Regression Model</i> | 877 |
| | 19.2.4 <i>The Stochastic Frontier Model</i> | 879 |
| 19.3 | Censored Data | 885 |
| | 19.3.1 <i>The Censored Normal Distribution</i> | 886 |

| | | |
|--------|---|-----|
| 19.3.2 | <i>The Censored Regression (Tobit) Model</i> | 888 |
| 19.3.3 | <i>Estimation</i> | 890 |
| 19.3.4 | <i>Two-Part Models and Corner Solutions</i> | 892 |
| 19.3.5 | <i>Some Issues in Specification</i> | 898 |
| | 19.3.5.a <i>Heteroscedasticity</i> | 898 |
| | 19.3.5.b <i>Nonnormality</i> | 899 |
| 19.3.6 | <i>Panel Data Applications</i> | 900 |
| 19.4 | <i>Models for Duration</i> | 901 |
| | 19.4.1 <i>Models for Duration Data</i> | 902 |
| | 19.4.2 <i>Duration Data</i> | 902 |
| | 19.4.3 <i>A Regression-Like Approach: Parametric Models of Duration</i> | 903 |
| | 19.4.3.a <i>Theoretical Background</i> | 903 |
| | 19.4.3.b <i>Models of the Hazard Function</i> | 904 |
| | 19.4.3.c <i>Maximum Likelihood Estimation</i> | 906 |
| | 19.4.3.d <i>Exogenous Variables</i> | 907 |
| | 19.4.3.e <i>Heterogeneity</i> | 908 |
| | 19.4.4 <i>Nonparametric and Semiparametric Approaches</i> | 909 |
| 19.5 | <i>Incidental Truncation and Sample Selection</i> | 912 |
| | 19.5.1 <i>Incidental Truncation in a Bivariate Distribution</i> | 913 |
| | 19.5.2 <i>Regression in a Model of Selection</i> | 913 |
| | 19.5.3 <i>Two-Step and Maximum Likelihood Estimation</i> | 916 |
| | 19.5.4 <i>Sample Selection in Nonlinear Models</i> | 920 |
| | 19.5.5 <i>Panel Data Applications of Sample Selection Models</i> | 923 |
| | 19.5.5.a <i>Common Effects in Sample Selection Models</i> | 924 |
| | 19.5.5.b <i>Attrition</i> | 926 |
| 19.6 | <i>Evaluating Treatment Effects</i> | 928 |
| | 19.6.1 <i>Regression Analysis of Treatment Effects</i> | 930 |
| | 19.6.1.a <i>The Normality Assumption</i> | 932 |
| | 19.6.1.b <i>Estimating the Effect of Treatment on the Treated</i> | 933 |
| | 19.6.2 <i>Propensity Score Matching</i> | 934 |
| | 19.6.3 <i>Regression Discontinuity</i> | 937 |
| 19.7 | <i>Summary and Conclusions</i> | 938 |

PART V Time Series and Macroeconometrics

CHAPTER 20 Serial Correlation 943

| | | |
|------|--|-----|
| 20.1 | <i>Introduction</i> | 943 |
| 20.2 | <i>The Analysis of Time-Series Data</i> | 946 |
| 20.3 | <i>Disturbance Processes</i> | 949 |
| | 20.3.1 <i>Characteristics of Disturbance Processes</i> | 949 |
| | 20.3.2 <i>AR(1) Disturbances</i> | 950 |

| | | |
|---------|--|-----|
| 20.4 | Some Asymptotic Results for Analyzing Time-Series Data | 952 |
| 20.4.1 | <i>Convergence of Moments—The Ergodic Theorem</i> | 953 |
| 20.4.2 | <i>Convergence to Normality—A Central Limit Theorem</i> | 955 |
| 20.5 | Least Squares Estimation | 958 |
| 20.5.1 | <i>Asymptotic Properties of Least Squares</i> | 958 |
| 20.5.2 | <i>Estimating the Variance of the Least Squares Estimator</i> | 959 |
| 20.6 | GMM Estimation | 961 |
| 20.7 | Testing for Autocorrelation | 962 |
| 20.7.1 | <i>Lagrange Multiplier Test</i> | 962 |
| 20.7.2 | <i>Box and Pierce’s Test and Ljung’s Refinement</i> | 962 |
| 20.7.3 | <i>The Durbin–Watson Test</i> | 963 |
| 20.7.4 | <i>Testing in the Presence of a Lagged Dependent Variable</i> | 963 |
| 20.7.5 | <i>Summary of Testing Procedures</i> | 964 |
| 20.8 | Efficient Estimation When Ω Is Known | 964 |
| 20.9 | Estimation When Ω Is Unknown | 966 |
| 20.9.1 | <i>AR(1) Disturbances</i> | 966 |
| 20.9.2 | <i>Application: Estimation of a Model with Autocorrelation</i> | 967 |
| 20.9.3 | <i>Estimation with a Lagged Dependent Variable</i> | 969 |
| 20.10 | Autoregressive Conditional Heteroscedasticity | 970 |
| 20.10.1 | <i>The ARCH(1) Model</i> | 971 |
| 20.10.2 | <i>ARCH(q), ARCH-in-Mean, and Generalized ARCH Models</i> | 972 |
| 20.10.3 | <i>Maximum Likelihood Estimation of the Garch Model</i> | 974 |
| 20.10.4 | <i>Testing for Garch Effects</i> | 976 |
| 20.10.5 | <i>Pseudo–Maximum Likelihood Estimation</i> | 977 |
| 20.11 | Summary and Conclusions | 979 |

CHAPTER 21 Nonstationary Data 982

| | | |
|--------|---|------|
| 21.1 | Introduction | 982 |
| 21.2 | Nonstationary Processes and Unit Roots | 982 |
| 21.2.1 | <i>Integrated Processes and Differencing</i> | 982 |
| 21.2.2 | <i>Random Walks, Trends, and Spurious Regressions</i> | 984 |
| 21.2.3 | <i>Tests for Unit Roots in Economic Data</i> | 987 |
| 21.2.4 | <i>The Dickey–Fuller Tests</i> | 988 |
| 21.2.5 | <i>The KPSS Test of Stationarity</i> | 998 |
| 21.3 | Cointegration | 999 |
| 21.3.1 | <i>Common Trends</i> | 1002 |
| 21.3.2 | <i>Error Correction and VAR Representations</i> | 1003 |
| 21.3.3 | <i>Testing for Cointegration</i> | 1005 |
| 21.3.4 | <i>Estimating Cointegration Relationships</i> | 1007 |
| 21.3.5 | <i>Application: German Money Demand</i> | 1007 |
| | 21.3.5.a <i>Cointegration Analysis and a Long-Run Theoretical Model</i> | 1008 |
| | 21.3.5.b <i>Testing for Model Instability</i> | 1009 |

| | | |
|------|--------------------------|------|
| 21.4 | Nonstationary Panel Data | 1010 |
| 21.5 | Summary and Conclusions | 1011 |

PART VI Appendices

| | | |
|-------------------|---|-------------|
| Appendix A | Matrix Algebra | 1013 |
| A.1 | Terminology | 1013 |
| A.2 | Algebraic Manipulation of Matrices | 1013 |
| A.2.1 | <i>Equality of Matrices</i> | 1013 |
| A.2.2 | <i>Transposition</i> | 1014 |
| A.2.3 | <i>Matrix Addition</i> | 1014 |
| A.2.4 | <i>Vector Multiplication</i> | 1015 |
| A.2.5 | <i>A Notation for Rows and Columns of a Matrix</i> | 1015 |
| A.2.6 | <i>Matrix Multiplication and Scalar Multiplication</i> | 1015 |
| A.2.7 | <i>Sums of Values</i> | 1017 |
| A.2.8 | <i>A Useful Idempotent Matrix</i> | 1018 |
| A.3 | Geometry of Matrices | 1019 |
| A.3.1 | <i>Vector Spaces</i> | 1019 |
| A.3.2 | <i>Linear Combinations of Vectors and Basis Vectors</i> | 1021 |
| A.3.3 | <i>Linear Dependence</i> | 1022 |
| A.3.4 | <i>Subspaces</i> | 1023 |
| A.3.5 | <i>Rank of a Matrix</i> | 1024 |
| A.3.6 | <i>Determinant of a Matrix</i> | 1026 |
| A.3.7 | <i>A Least Squares Problem</i> | 1027 |
| A.4 | Solution of a System of Linear Equations | 1029 |
| A.4.1 | <i>Systems of Linear Equations</i> | 1029 |
| A.4.2 | <i>Inverse Matrices</i> | 1030 |
| A.4.3 | <i>Nonhomogeneous Systems of Equations</i> | 1032 |
| A.4.4 | <i>Solving the Least Squares Problem</i> | 1032 |
| A.5 | Partitioned Matrices | 1032 |
| A.5.1 | <i>Addition and Multiplication of Partitioned Matrices</i> | 1033 |
| A.5.2 | <i>Determinants of Partitioned Matrices</i> | 1033 |
| A.5.3 | <i>Inverses of Partitioned Matrices</i> | 1033 |
| A.5.4 | <i>Deviations from Means</i> | 1034 |
| A.5.5 | <i>Kronecker Products</i> | 1034 |
| A.6 | Characteristic Roots and Vectors | 1035 |
| A.6.1 | <i>The Characteristic Equation</i> | 1035 |
| A.6.2 | <i>Characteristic Vectors</i> | 1036 |
| A.6.3 | <i>General Results for Characteristic Roots and Vectors</i> | 1036 |
| A.6.4 | <i>Diagonalization and Spectral Decomposition of a Matrix</i> | 1037 |
| A.6.5 | <i>Rank of a Matrix</i> | 1037 |
| A.6.6 | <i>Condition Number of a Matrix</i> | 1039 |
| A.6.7 | <i>Trace of a Matrix</i> | 1039 |
| A.6.8 | <i>Determinant of a Matrix</i> | 1040 |
| A.6.9 | <i>Powers of a Matrix</i> | 1040 |
| A.6.10 | <i>Idempotent Matrices</i> | 1042 |

| | | | |
|-------------------|--------|---|-------------|
| | A.6.11 | <i>Factoring a Matrix</i> | 1042 |
| | A.6.12 | <i>The Generalized Inverse of a Matrix</i> | 1043 |
| A.7 | | Quadratic Forms and Definite Matrices | 1044 |
| | A.7.1 | <i>Nonnegative Definite Matrices</i> | 1045 |
| | A.7.2 | <i>Idempotent Quadratic Forms</i> | 1046 |
| | A.7.3 | <i>Comparing Matrices</i> | 1046 |
| A.8 | | Calculus and Matrix Algebra | 1047 |
| | A.8.1 | <i>Differentiation and the Taylor Series</i> | 1047 |
| | A.8.2 | <i>Optimization</i> | 1050 |
| | A.8.3 | <i>Constrained Optimization</i> | 1052 |
| | A.8.4 | <i>Transformations</i> | 1054 |
| Appendix B | | Probability and Distribution Theory | 1055 |
| B.1 | | Introduction | 1055 |
| B.2 | | Random Variables | 1055 |
| | B.2.1 | <i>Probability Distributions</i> | 1055 |
| | B.2.2 | <i>Cumulative Distribution Function</i> | 1056 |
| B.3 | | Expectations of a Random Variable | 1057 |
| B.4 | | Some Specific Probability Distributions | 1059 |
| | B.4.1 | <i>The Normal Distribution</i> | 1059 |
| | B.4.2 | <i>The Chi-Squared, t, and F Distributions</i> | 1061 |
| | B.4.3 | <i>Distributions with Large Degrees of Freedom</i> | 1063 |
| | B.4.4 | <i>Size Distributions: The Lognormal Distribution</i> | 1064 |
| | B.4.5 | <i>The Gamma and Exponential Distributions</i> | 1064 |
| | B.4.6 | <i>The Beta Distribution</i> | 1065 |
| | B.4.7 | <i>The Logistic Distribution</i> | 1065 |
| | B.4.8 | <i>The Wishart Distribution</i> | 1065 |
| | B.4.9 | <i>Discrete Random Variables</i> | 1066 |
| B.5 | | The Distribution of a Function of a Random Variable | 1066 |
| B.6 | | Representations of a Probability Distribution | 1068 |
| B.7 | | Joint Distributions | 1070 |
| | B.7.1 | <i>Marginal Distributions</i> | 1070 |
| | B.7.2 | <i>Expectations in a Joint Distribution</i> | 1071 |
| | B.7.3 | <i>Covariance and Correlation</i> | 1071 |
| | B.7.4 | <i>Distribution of a Function of Bivariate Random Variables</i> | 1072 |
| B.8 | | Conditioning in a Bivariate Distribution | 1074 |
| | B.8.1 | <i>Regression: The Conditional Mean</i> | 1074 |
| | B.8.2 | <i>Conditional Variance</i> | 1075 |
| | B.8.3 | <i>Relationships Among Marginal and Conditional Moments</i> | 1075 |
| | B.8.4 | <i>The Analysis of Variance</i> | 1077 |
| B.9 | | The Bivariate Normal Distribution | 1077 |
| B.10 | | Multivariate Distributions | 1078 |
| | B.10.1 | <i>Moments</i> | 1078 |

| | | | |
|-------------------|---------------|---|-------------|
| | <i>B.10.2</i> | <i>Sets of Linear Functions</i> | 1079 |
| | <i>B.10.3</i> | <i>Nonlinear Functions</i> | 1080 |
| B.11 | | The Multivariate Normal Distribution | 1081 |
| | <i>B.11.1</i> | <i>Marginal and Conditional Normal Distributions</i> | 1081 |
| | <i>B.11.2</i> | <i>The Classical Normal Linear Regression Model</i> | 1082 |
| | <i>B.11.3</i> | <i>Linear Functions of a Normal Vector</i> | 1083 |
| | <i>B.11.4</i> | <i>Quadratic forms in a Standard Normal Vector</i> | 1083 |
| | <i>B.11.5</i> | <i>The F Distribution</i> | 1085 |
| | <i>B.11.6</i> | <i>A Full Rank Quadratic Form</i> | 1085 |
| | <i>B.11.7</i> | <i>Independence of a Linear and a Quadratic Form</i> | 1086 |
| Appendix C | | Estimation and Inference | 1087 |
| | C.1 | Introduction | 1087 |
| | C.2 | Samples and Random Sampling | 1088 |
| | C.3 | Descriptive Statistics | 1088 |
| | C.4 | Statistics as Estimators—Sampling Distributions | 1091 |
| | C.5 | Point Estimation of Parameters | 1095 |
| | | <i>C.5.1 Estimation in a Finite Sample</i> | 1095 |
| | | <i>C.5.2 Efficient Unbiased Estimation</i> | 1098 |
| | C.6 | Interval Estimation | 1100 |
| | C.7 | Hypothesis Testing | 1102 |
| | | <i>C.7.1 Classical Testing Procedures</i> | 1102 |
| | | <i>C.7.2 Tests Based on Confidence Intervals</i> | 1105 |
| | | <i>C.7.3 Specification Tests</i> | 1106 |
| Appendix D | | Large-Sample Distribution Theory | 1106 |
| | D.1 | Introduction | 1106 |
| | D.2 | Large-Sample Distribution Theory | 1107 |
| | | <i>D.2.1 Convergence in Probability</i> | 1107 |
| | | <i>D.2.2 Other forms of Convergence and Laws of Large Numbers</i> | 1110 |
| | | <i>D.2.3 Convergence of Functions</i> | 1113 |
| | | <i>D.2.4 Convergence to a Random Variable</i> | 1114 |
| | | <i>D.2.5 Convergence in Distribution: Limiting Distributions</i> | 1116 |
| | | <i>D.2.6 Central Limit Theorems</i> | 1118 |
| | | <i>D.2.7 The Delta Method</i> | 1123 |
| | D.3 | Asymptotic Distributions | 1124 |
| | | <i>D.3.1 Asymptotic Distribution of a Nonlinear Function</i> | 1126 |
| | | <i>D.3.2 Asymptotic Expectations</i> | 1127 |
| | D.4 | Sequences and the Order of a Sequence | 1128 |
| Appendix E | | Computation and Optimization | 1129 |
| | E.1 | Introduction | 1129 |
| | E.2 | Computation in Econometrics | 1130 |
| | | <i>E.2.1 Computing Integrals</i> | 1130 |

24 Contents

| | | | |
|--|--------------|---|-------------|
| | <i>E.2.2</i> | <i>The Standard Normal Cumulative Distribution Function</i> | 1130 |
| | <i>E.2.3</i> | <i>The Gamma and Related Functions</i> | 1131 |
| | <i>E.2.4</i> | <i>Approximating Integrals by Quadrature</i> | 1132 |
| E.3 | | Optimization | 1133 |
| | <i>E.3.1</i> | <i>Algorithms</i> | 1135 |
| | <i>E.3.2</i> | <i>Computing Derivatives</i> | 1136 |
| | <i>E.3.3</i> | <i>Gradient Methods</i> | 1137 |
| | <i>E.3.4</i> | <i>Aspects of Maximum Likelihood Estimation</i> | 1140 |
| | <i>E.3.5</i> | <i>Optimization with Constraints</i> | 1141 |
| | <i>E.3.6</i> | <i>Some Practical Considerations</i> | 1142 |
| | <i>E.3.7</i> | <i>The EM Algorithm</i> | 1144 |
| E.4 | | Examples | 1146 |
| | <i>E.4.1</i> | <i>Function of One Parameter</i> | 1146 |
| | <i>E.4.2</i> | <i>Function of Two Parameters: The Gamma Distribution</i> | 1147 |
| | <i>E.4.3</i> | <i>A Concentrated Log-Likelihood Function</i> | 1148 |
| Appendix F | | Data Sets Used in Applications | 1149 |
| References | | | 1155 |
| Combined Author and Subject Index | | | 1201 |