Contents

eface		•
Dire	ected and undirected graphs	
1.1	Formal description of graphs	1
1.2		4
1.3		5
1.4		7
1.5		11
1.6	• • • • • • • • • • • • • • • • • • • •	14
	* * * *	14
		15
1.7		18
1.8	· · · · · · · · · · · · · · · · · · ·	24
Gra	phs and matrices	26
2.1	Adjacency matrix	26
	Isomorphic graphs and the adjacency matrix	28
	Components and the adjacency matrix	29
	Adjacency list	30
2.2	Incidence matrix	30
2.3	Distances in graphs	31
	The adjacency matrix and paths	32
	The adjacency matrix, the distance matrix and circuits	33
2.4	Endomorphisms and commuting graphs	34
2.5	The characteristic polynomial and eigenvalues	35
2.6	Circulant graphs	40
2.7		43
	-	43
		44
		45
2.8	Comments	46
Cot	ogories and functors	48
		48
3.1		
	Constructs and small and large categories	45
	1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 Gra 2.1 2.2 2.3 2.4 2.5 2.6 2.7	Directed and undirected graphs 1.1 Formal description of graphs 1.2 Connectedness and equivalence relations 1.3 Some special graphs 1.4 Homomorphisms 1.5 Half-, locally, quasi-strong and metric homomorphisms 1.6 The factor graph, congruences, and the Homomorphism Theorem Factor graphs The Homomorphism Theorem 1.7 The endomorphism type of a graph 1.8 Comments Graphs and matrices 2.1 Adjacency matrix Isomorphic graphs and the adjacency matrix Components and the adjacency matrix Adjacency list 2.2 Incidence matrix 2.3 Distances in graphs The adjacency matrix, the distance matrix and circuits 2.4 Endomorphisms and commuting graphs 2.5 The characteristic polynomial and eigenvalues 2.6 Circulant graphs Eigenvalues and the combinatorial structure Cospectral graphs Eigenvalues, diameter and regularity Automorphisms and eigenvalues 2.8 Comments Categories and functors 3.1 Categories Categories with sets and mappings, I



xii Contents

		Special objects and morphisms
		Categories with sets and mappings, II
		Categories with graphs
		Other categories
	3.2	Products & Co
		Coproducts
		Products
		Tensor products
		Categories with sets and mappings, III
	3.3	Functors
	0.0	Covariant and contravariant functors
		Composition of functors
		Special functors – examples
		Mor functors
		Properties of functors
	3.4	Comments
	J. 4	Comments
4	Bina	ary graph operations 64
	4.1	Unions
		The union
		The join
		The edge sum
	4.2	Products
		The cross product
		The coamalgamated product
		The disjunction of graphs
	4.3	Tensor products and the product in <i>EGra</i>
		The box product
		The boxcross product
		The complete product
		Synopsis of the results
		Product constructions as functors in one variable
	4.4	Lexicographic products and the corona
		Lexicographic products
		The corona
	4.5	Algebraic properties
	4.6	Mor constructions
		Diamond products
		Left inverses for tensor functors
		Power products
		Left inverses to product functors
	4.7	Comments

Contents xiii

5	Line	graph and other unary graph operations	91
	5.1	Complements, opposite graphs and geometric duals	91
	5.2	The line graph	92
		Determinability of G by LG	95
	5.3	Spectra of line graphs	97
		Which graphs are line graphs?	99
	5.4	The total graph	101
	5.5	The tree graph	
	5.6	Comments	
6	Gra	phs and vector spaces	104
	6.1	Vertex space and edge space	
		The boundary & Co	
		Matrix representation	
	6.2	Cycle spaces, bases & Co	
		The cycle space	
		The cocycle space	
		Orthogonality	
		The boundary operator & Co	
	6.3	Application: MacLane's planarity criterion	
	6.4	Homology of graphs	
		Exact sequences of vector spaces	
		Chain complexes and homology groups of graphs	
	6.5	Application: number of spanning trees	
	6.6	Application: electrical networks	
	6.7	Application: squared rectangles	128
	6.8	Application: shortest (longest) paths	132
	6.9	Comments	
7	Gra	phs, groups and monoids	136
	7.1	Groups of a graph	136
		Edge group	137
	7.2	Asymmetric graphs and rigid graphs	138
	7.3	Cayley graphs	
	7.4	Frucht-type results	146
		Frucht's theorem and its generalization for monoids	147
	7.5	Graph-theoretic requirements	148
		Smallest graphs for given groups	149
		Additional properties of group-realizing graphs	150
	7.6	Transformation monoids and permutation groups	
	7.7	Actions on graphs	
		Fixed-point-free actions on graphs	

xiv Contents

		Transitive actions on graphs	
	7.8	Comments	
8	The	characteristic polynomial of graphs	161
	8.1	Eigenvectors of symmetric matrices	161
		Eigenvalues and connectedness	
		Regular graphs and eigenvalues	
	8.2	Interpretation of the coefficients of chapo(G)	
		Interpretation of the coefficients for undirected graphs	
	8.3	Spectra of trees	
		Recursion formula for trees	
	8.4	The spectral radius of undirected graphs	
		Subgraphs	
		Upper bounds	
		Lower bounds	
	8.5	Spectral determinability	
		Spectral uniqueness of K_n and $K_{p,q}$	
	8.6	Eigenvalues and group actions	
		Groups, orbits and eigenvalues	
	8.7	Transitive graphs and eigenvalues	
	0.7	Derogatory graphs	
		Graphs with Abelian groups	
	8.8	Comments	
9	Gra	phs and monoids	181
	9.1	Semigroups	181
	9.2	End-regular bipartite graphs	
		Regular endomorphisms and retracts	
		End-regular and End-orthodox connected bipartite graphs	
	9.3	Locally strong endomorphisms of paths	
		Undirected paths	
		Directed paths	
		Algebraic properties of LEnd	
	9.4	Wreath product of monoids over an act	
	9.5	Structure of the strong monoid	
	,,,	The canonical strong decomposition of G	
		Decomposition of SEnd	201
		A generalized wreath product with a small category	
		Cardinality of SEnd(G)	
	9.6	Some algebraic properties of SEnd	204
	7.0	Regularity and more for T_A	

Contents xv

		Regularity and more for $SEnd(G)$
	9.7	Comments
10	Com	positions, unretractivities and monoids 208
	10.1	Lexicographic products
		Unretractivities and lexicographic products
	10.3	Monoids and lexicographic products
	10.4	The union and the join
		The sum of monoids
		The sum of endomorphism monoids
		Unretractivities
	10.5	The box product and the cross product
		Unretractivities
		The product of endomorphism monoids
	10.6	Comments
11	•	ey graphs of semigroups 225
	11.1	The Cay functor
		Reflection and preservation of morphisms
		Does Cay produce strong homomorphisms?
	11.2	Products and equalizers
		Categorical products
		Equalizers
		Other product constructions
		Cayley graphs of right and left groups
		Cayley graphs of strong semilattices of semigroups
		Application: strong semilattices of (right or left) groups 240
	11.6	Comments
12	Vert	ex transitive Cayley graphs 245
		Aut-vertex transitivity
		Application to strong semilattices of right groups
		ColAut(S , C)-vertex transitivity
		Aut(S, C)-vertex transitivity
	12.3	Application to strong semilattices of left groups
		Application to strong semilattices of groups
	12.4	$\operatorname{End}'(S,C)$ -vertex transitive Cayley graphs
		Comments
12	10. ·	
13		eddings of Cayley graphs – genus of semigroups 261
		The genus of a group
	13.2	Toroidal right groups

•	~
KV1	Contents
* * *	

13.3 The genus of $(A \times R_r)$	
Constructions of Cayley graphs for $A \times R_2$ and $A \times R_3 \dots \dots$	270
13.4 Non-planar Clifford semigroups	275
13.5 Planar Clifford semigroups	279
13.6 Comments	
Bibliography	285
Index	301
Index of symbols	307