

## Contents

**Preface** *XIII*

**Table of Contents and Categories** *XV*

**Constants, Signs, Symbols, and General Remarks** *XVII*

<b>1</b>	<b>Vectors</b>	<i>1</i>
1.1	Definition and Important Properties	<i>1</i>
1.1.1	Definitions	<i>1</i>
1.2	Product of a Scalar and a Vector	<i>2</i>
1.3	Position Vector	<i>2</i>
1.4	Scalar Product	<i>3</i>
1.5	Vector Product	<i>4</i>
1.6	Differentiation	<i>6</i>
1.7	Spherical Coordinates	<i>6</i>
1.8	Cylindrical Coordinates	<i>7</i>
<b>2</b>	<b>Tensors and Matrices</b>	<i>11</i>
2.1	Dyadic or Tensor Product	<i>11</i>
2.2	Cartesian Representation	<i>12</i>
2.3	Dot Product	<i>13</i>
2.3.1	Unit Tensor	<i>14</i>
2.4	Symmetric Tensor	<i>15</i>
2.5	Eigenvalue Problem	<i>15</i>
<b>3</b>	<b>Hamiltonian Mechanics</b>	<i>17</i>
3.1	Newtonian, Lagrangian and Hamiltonian Descriptions	<i>17</i>
3.1.1	Newtonian Description	<i>17</i>
3.1.2	Lagrangian Description	<i>19</i>
3.1.3	Hamiltonian Description	<i>20</i>
3.2	State of Motion in Phase Space. Reversible Motion	<i>22</i>
3.3	Hamiltonian for a System of many Particles	<i>28</i>
3.4	Canonical Transformation	<i>32</i>
3.5	Poisson Brackets	<i>36</i>

<b>4</b>	<b>Coupled Oscillators and Normal Modes</b>	<b>43</b>
4.1	Oscillations of Particles on a String and Normal Modes	43
4.2	Normal Coordinates	48
<b>5</b>	<b>Stretched String</b>	<b>53</b>
5.1	Transverse Oscillations of a Stretched String	53
5.2	Normal Coordinates for a String	58
<b>6</b>	<b>Vector Calculus and the del Operator</b>	<b>65</b>
6.1	Differentiation in Time	65
6.2	Space Derivatives	66
6.2.1	The Gradient	66
6.2.2	The Divergence	67
6.2.3	The Curl	67
6.2.4	Space Derivatives of Products	67
6.3	Space Derivatives in Curvilinear Coordinates	69
6.3.1	Spherical Coordinates $(r, \theta, \varphi)$	69
6.3.2	Cylindrical Coordinates $(\rho, \varphi, z)$	69
6.4	Integral Theorems	70
6.4.1	The Line Integral of $\nabla\phi$	70
6.4.2	Stokes's Theorem	70
6.5	Gauss's Theorem	72
6.6	Derivation of the Gradient, Divergence and Curl	73
<b>7</b>	<b>Electromagnetic Waves</b>	<b>77</b>
7.1	Electric and Magnetic Fields in a Vacuum	77
7.2	The Electromagnetic Field Theory	82
<b>8</b>	<b>Fluid Dynamics</b>	<b>87</b>
8.1	Continuity Equation	87
8.2	Fluid Equation of Motion	89
8.3	Fluid Dynamics and Statistical Mechanics	92
<b>9</b>	<b>Irreversible Processes</b>	<b>97</b>
9.1	Irreversible Phenomena, Viscous Flow, Diffusion	97
9.2	Collision Rate and Mean Free Path	100
9.3	Ohm's Law, Conductivity, and Matthiessen's Rule	103
<b>10</b>	<b>The Entropy</b>	<b>107</b>
10.1	Foundations of Thermodynamics	107
10.2	The Carnot Cycle	109
10.3	Carnot's Theorem	111
10.4	Heat Engines and Refrigerating Machines	113
10.5	Clausius's Theorem	114
10.6	The Entropy	119
10.7	The Exact Differential	122

- 11 Thermodynamic Inequalities 125**
  - 11.1 Irreversible Processes and the Entropy 125
  - 11.2 The Helmholtz Free Energy 128
  - 11.3 The Gibbs Free Energy 130
  - 11.4 Maxwell Relations 132
  - 11.5 Heat Capacities 136
  - 11.6 Nonnegative Heat Capacity and Compressibility 140
- 12 Probability, Statistics and Density 147**
  - 12.1 Probabilities 147
  - 12.2 Binomial Distribution 150
  - 12.3 Average and Root-Mean-Square Deviation. Random Walks 153
  - 12.4 Microscopic Number Density 156
  - 12.5 Dirac's Delta Function 160
  - 12.6 The Three-Dimensional Delta Function 161
- 13 Liouville Equation 165**
  - 13.1 Liouville's Theorem 165
  - 13.2 Probability Distribution Function. The Liouville Equation 169
  - 13.3 The Gibbs Ensemble 173
  - 13.4 Many Particles Moving in Three Dimensions 175
  - 13.5 More about the Liouville Equation 177
  - 13.6 Symmetries of Hamiltonians and Stationary States 179
- 14 Generalized Vectors and Linear Operators 183**
  - 14.1 Generalized Vectors. Matrices 183
  - 14.2 Linear Operators 188
  - 14.3 The Eigenvalue Problem 192
  - 14.4 Orthogonal Representation 196
- 15 Quantum Mechanics for a Particle 201**
  - 15.1 Quantum Description of a Linear Motion 201
  - 15.2 The Momentum Eigenvalue Problem 207
  - 15.3 The Energy Eigenvalue Problem 211
- 16 Fourier Series and Transforms 213**
  - 16.1 Fourier Series 213
  - 16.2 Fourier Transforms 214
  - 16.3 Bra and Ket Notations 215
  - 16.4 Heisenberg's Uncertainty Principle 217
- 17 Quantum Angular Momentum 221**
  - 17.1 Quantum Angular Momentum 221
  - 17.2 Properties of Angular Momentum 224
- 18 Spin Angular Momentum 229**
  - 18.1 The Spin Angular Momentum 229
  - 18.2 The Spin of the Electron 231

- 18.3 The Magnetogyric Ratio 234
  - 18.3.1 A. Free Electron 235
  - 18.3.2 B. Free Proton 236
  - 18.3.3 C. Free Neutron 237
  - 18.3.4 D. Atomic Nuclei 237
  - 18.3.5 E. Atoms and Ions 237
- 19 Time-Dependent Perturbation Theory 239**
  - 19.1 Perturbation Theory 1; The Dirac Picture 239
  - 19.2 Scattering Problem; Fermi's Golden Rule 242
  - 19.3 Perturbation Theory 2. Second Intermediate Picture 245
- 20 Laplace Transformation 249**
  - 20.1 Laplace Transformation 249
  - 20.2 The Electric Circuit Equation 251
  - 20.3 Convolution Theorem 252
  - 20.4 Linear Operator Algebras 253
- 21 Quantum Harmonic Oscillator 255**
  - 21.1 Energy Eigenvalues 255
  - 21.2 Quantum Harmonic Oscillator 259
- 22 Permutation Group 263**
  - 22.1 Permutation Group 263
  - 22.2 Odd and Even Permutations 267
- 23 Quantum Statistics 273**
  - 23.1 Classical Indistinguishable Particles 273
  - 23.2 Quantum-Statistical Postulate. Symmetric States for Bosons 276
  - 23.3 Antisymmetric States for Fermions. Pauli's Exclusion Principle 278
  - 23.4 Occupation-Number Representation 280
- 24 The Free-Electron Model 283**
  - 24.1 Free Electrons and the Fermi Energy 283
  - 24.2 Density of States 287
  - 24.3 Qualitative Discussion 291
  - 24.4 Sommerfeld's Calculations 293
- 25 The Bose-Einstein Condensation 297**
  - 25.1 Liquid Helium 297
  - 25.2 The Bose-Einstein Condensation of Free Bosons 298
  - 25.3 Bosons in Condensed Phase 301
- 26 Magnetic Susceptibility 307**
  - 26.1 Introduction 307
  - 26.2 Pauli Paramagnetism 308
  - 26.3 Motion of a Charged Particle in Electromagnetic Fields 310
  - 26.4 Electromagnetic Potentials 313
  - 26.5 The Landau States and Energies 316

- 26.6 The Degeneracy of the Landau Levels 317
- 26.7 Landau Diamagnetism 321
- 27 Theory of Variations 329**
- 27.1 The Euler–Lagrange Equation 329
- 27.2 Fermat’s Principle 331
- 27.3 Hamilton’s Principle 333
- 27.4 Lagrange’s Field Equation 333
- 28 Second Quantization 335**
- 28.1 Boson Creation and Annihilation Operators 335
- 28.2 Observables 338
- 28.3 Fermions Creation and Annihilation Operators 340
- 28.4 Heisenberg Equation of Motion 341
- 29 Quantum Statistics of Composites 345**
- 29.1 Ehrenfest–Oppenheimer–Bethe’s Rule 345
- 29.2 Two-Particle Composites 346
- 29.3 Discussion 351
- 30 Superconductivity 357**
- 30.1 Basic Properties of a Superconductor 357
- 30.1.1 Zero Resistance 357
- 30.1.2 Meissner Effect 357
- 30.1.3 Ring Supercurrent and Flux Quantization 359
- 30.1.4 Josephson Effects 360
- 30.1.5 Energy Gap 362
- 30.1.6 Sharp Phase Change 363
- 30.2 Occurrence of a Superconductor 363
- 30.2.1 Elemental Superconductors 363
- 30.2.2 Compound Superconductors 363
- 30.2.3 High- $T_c$  Superconductors 365
- 30.3 Theoretical Survey 365
- 30.3.1 The Cause of Superconductivity 365
- 30.3.2 The Bardeen–Cooper–Schrieffer Theory 366
- 30.4 Quantum-Statistical Theory 367
- 30.4.1 The Full Hamiltonian 367
- 30.4.2 Summary of the Results 369
- 31 Complex Numbers and Taylor Series 375**
- 31.1 Complex Numbers 375
- 31.2 Exponential and Logarithmic Functions 377
- 31.2.1 Laws of Exponents 377
- 31.2.2 Natural Logarithm 377
- 31.2.3 Relationship between Exponential and Trigonometric Functions 377
- 31.3 Hyperbolic Functions 378
- 31.3.1 Definition of Hyperbolic Functions 378

31.3.2	Addition Formulas	378
31.3.3	Double-Angle Formulas	379
31.3.4	Sum, Difference and Product of Hyperbolic Functions	379
31.3.5	Relationship between Hyperbolic and Trigonometric Functions	379
31.4	Taylor Series	380
31.4.1	Derivatives	380
31.4.2	Taylor Series	381
31.4.3	Binomial Series	381
31.4.4	Series for Exponential and Logarithmic Functions	382
31.5	Convergence of a Series	382
<b>32</b>	<b>Analyticity and Cauchy–Riemann Equations</b>	<b>385</b>
32.1	The Analytic Function	385
32.2	Poles	387
32.3	Exponential Functions	387
32.4	Branch Points	388
32.5	Function with Continuous Singularities	389
32.6	Cauchy–Riemann Relations	390
32.7	Cauchy–Riemann Relations Applications	391
<b>33</b>	<b>Cauchy’s Fundamental Theorem</b>	<b>395</b>
33.1	Cauchy’s Fundamental Theorem	395
33.2	Line Integrals	398
33.3	Circular Integrals	400
33.4	Cauchy’s Integral Formula	402
<b>34</b>	<b>Laurent Series</b>	<b>405</b>
34.1	Taylor Series and Convergence Radius	405
34.2	Uniform Convergence	406
34.3	Laurent Series	407
<b>35</b>	<b>Multivalued Functions</b>	<b>411</b>
35.1	Square-Root Functions. Riemann Sheets and Cut	411
35.2	Multivalued Functions	413
<b>36</b>	<b>Residue Theorem and Its Applications</b>	<b>415</b>
36.1	Residue Theorem	415
36.2	Integrals of the Form $\int_{-\infty}^{\infty} dx f(x)$	417
36.3	Integrals of the Type $\int_{-\infty}^{\infty} dx e^{ix} f(x)$	419
36.4	Integrals of the Type $\int_0^{2\pi} d\theta f(\cos \theta, \sin \theta)$	420
36.5	Miscellaneous Integrals	421

<b>Appendix A Representation-Independence of Poisson Brackets</b>	<b>423</b>
<b>Appendix B Proof of the Convolution Theorem</b>	<b>427</b>
<b>Appendix C Statistical Weight for the Landau States</b>	<b>431</b>
<b>Appendix D Useful Formulas</b>	<b>433</b>
<b>References</b>	<b>435</b>
<b>Index</b>	<b>439</b>